Implementation Security In Cryptography

Lecture 11: Entering the World of Attacks

Recap

- In the last lecture
  - Compact design of AES

Today

- Few more words about implementations
- Entering the world of attacks.

AES Once Again

- Well, we have seen how it is done in hardware..
- But what about software?
- A popular, extremely fast, yet terrible approach — T tables
  - Used quite a lot in OpenSSL
    - Not used anymore due to several **cache timing attacks**
- But table based secure implementations also do exist

AES T Tables

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

- Let us consider the MixColumns operation
- In software, each 32-bit AES column is a uint32 variable.

$$s_0 = 2c_0 \oplus 3c_1 \oplus 1c_2 \oplus 1c_3$$
$$s_1 = 1c_0 \oplus 2c_1 \oplus 3c_2 \oplus 1c_3$$
$$s_2 = 1c_0 \oplus 1c_1 \oplus 2c_2 \oplus 3c_3$$
$$s_3 = 3c_0 \oplus 1c_1 \oplus 1c_2 \oplus 2c_3$$

AES T Tables

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- Let us consider the MixColumns operation
- In software, each 32-bit AES column is a uint32 variable.
- So we can store s in a uint32 variable.
- Here each s_i is 8-bit, so we can simply do concatenation to get 32-bits.

$$s_0 = 2c_0 \oplus 3c_1 \oplus 1c_2 \oplus 1c_3$$
$$s_1 = 1c_0 \oplus 2c_1 \oplus 3c_2 \oplus 1c_3$$
$$s_2 = 1c_0 \oplus 1c_1 \oplus 2c_2 \oplus 3c_3$$
$$s_3 = 3c_0 \oplus 1c_1 \oplus 1c_2 \oplus 2c_3$$

```
s = s0 | s1 | s2 | s3
s = 2*c0 ^ 3*c1 ^ 1*c2 ^ 1*c3 |
    1*c0 ^ 2*c1 ^ 3*c2 ^ 1*c3 |
    1*c0 ^ 1*c1 ^ 2*c2 ^ 3*c3 |
    3*c0 ^ 1*c1 ^ 1*c2 ^ 2*c3
```

AES T Tables

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

- Here each s_i is 8-bit, so we can simply do concatenation to get 32-bits.
- Now we can also rearrange the terms.
  - Why this is beneficial??

```
s = s0 | s1 | s2 | s3
s = 2*c0 ^ 3*c1 ^ 1*c2 ^ 1*c3 |
    1*c0 ^ 2*c1 ^ 3*c2 ^ 1*c3 |
    1*c0 ^ 1*c1 ^ 2*c2 ^ 3*c3 |
    3*c0 ^ 1*c1 ^ 1*c2 ^ 2*c3
```

```
s = (2*c0 | 1*c0 | 1*c0 | 3*c0) ^
    (3*c1 | 2*c1 | 1*c1 | 1*c1) ^
    (1*c2 | 3*c2 | 2*c2 | 1*c2) ^
    (1*c3 | 1*c3 | 3*c3 | 2*c3)
```

AES T Tables

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

- Here each s_i is 8-bit, so we can simply do concatenation to get 32-bits.
- Now we can also rearrange the terms.
  - Why this is beneficial??
  - Observe that you can compute each term being XORed only from a c_i

```
s = s0 | s1 | s2 | s3
s = 2*c0 ^ 3*c1 ^ 1*c2 ^ 1*c3 |
    1*c0 ^ 2*c1 ^ 3*c2 ^ 1*c3 |
    1*c0 ^ 1*c1 ^ 2*c2 ^ 3*c3 |
    3*c0 ^ 1*c1 ^ 1*c2 ^ 2*c3
```

```
s = (2*c0 | 1*c0 | 1*c0 | 3*c0) ^
    (3*c1 | 2*c1 | 1*c1 | 1*c1) ^
    (1*c2 | 3*c2 | 2*c2 | 1*c2) ^
    (1*c3 | 1*c3 | 3*c3 | 2*c3)
```

AES T Tables

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

- Why this is beneficial??
  - Observe that you can compute each term being XORed only from a c_i
  - Since c_i is 8-bit, so we can compute all possible 256 values and store them in. A table.
  - Same can be done for all c_i
    - One table for each
  - **Catch**: Table lookup is much faster than a finite field operation.

```
s = s0 | s1 | s2 | s3
s = 2*c0 ^ 3*c1 ^ 1*c2 ^ 1*c3 |
    1*c0 ^ 2*c1 ^ 3*c2 ^ 1*c3 |
    1*c0 ^ 1*c1 ^ 2*c2 ^ 3*c3 |
    3*c0 ^ 1*c1 ^ 1*c2 ^ 2*c3
```

```
s = (2*c0 | 1*c0 | 1*c0 | 3*c0) ^
    (3*c1 | 2*c1 | 1*c1 | 1*c1) ^
    (1*c2 | 3*c2 | 2*c2 | 1*c2) ^
    (1*c3 | 1*c3 | 3*c3 | 2*c3)
```

AES T Tables

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

- Why this is beneficial??
  - We denote these tables as te0, te1, te2, and te3.
  - The operation is as follows:

```
te0[i] = 2*i | 1*i | 1*i | 3*i
te1[i] = 3*i | 2*i | 1*i | 1*i
te2[i] = 1*i | 3*i | 2*i | 1*i
te3[i] = 1*i | 1*i | 3*i | 2*i

s = te0[c0] ^ te1[c1] ^ te2[c2] ^ te3[c3]
```

```
s = s0 | s1 | s2 | s3
s = 2*c0 ^ 3*c1 ^ 1*c2 ^ 1*c3 |
    1*c0 ^ 2*c1 ^ 3*c2 ^ 1*c3 |
    1*c0 ^ 1*c1 ^ 2*c2 ^ 3*c3 |
    3*c0 ^ 1*c1 ^ 1*c2 ^ 2*c3
```

```
s = (2*c0 | 1*c0 | 1*c0 | 3*c0) ^
    (3*c1 | 2*c1 | 1*c1 | 1*c1) ^
    (1*c2 | 3*c2 | 2*c2 | 1*c2) ^
    (1*c3 | 1*c3 | 3*c3 | 2*c3)
```

It does not ends here…

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

```
s = s0 | s1 | s2 | s3
s = 2*c0 ^ 3*c1 ^ 1*c2 ^ 1*c3 |
    1*c0 ^ 2*c1 ^ 3*c2 ^ 1*c3 |
    1*c0 ^ 1*c1 ^ 2*c2 ^ 3*c3 |
    3*c0 ^ 1*c1 ^ 1*c2 ^ 2*c3
```

- Can we do better than this?
-

```
te0[i] = 2*i | 1*i | 1*i | 3*i
te1[i] = 3*i | 2*i | 1*i | 1*i
te2[i] = 1*i | 3*i | 2*i | 1*i
te3[i] = 1*i | 1*i | 3*i | 2*i

s = te0[c0] ^ te1[c1] ^ te2[c2] ^ te3[c3]
```

```
s = (2*c0 | 1*c0 | 1*c0 | 3*c0) ^
    (3*c1 | 2*c1 | 1*c1 | 1*c1) ^
    (1*c2 | 3*c2 | 2*c2 | 1*c2) ^
    (1*c3 | 1*c3 | 3*c3 | 2*c3)
```

It does not ends here…

```
s = (2*c0 | 1*c0 | 1*c0 | 3*c0) ^
    (3*c1 | 2*c1 | 1*c1 | 1*c1) ^
    (1*c2 | 3*c2 | 2*c2 | 1*c2) ^
    (1*c3 | 1*c3 | 3*c3 | 2*c3)
```

- Can we do better than this?
- Observe that: $c0|c1|c2|c3$ can be
  - `S[b0]|S[b5]|S[b10]|S[b15]` or, `S[b4]|S[b9]|S[b14]|S[b3]` or `S[b8]|S[b13]|S[b2]|S[b7]` `S[b12]|S[b1]|S[b6]|S[b11]`
  - So we can merge subtypes and shift rows in a table

```
R0 = (2*S[b0]  | 1*S[b0]  | 1*S[b0]  | 3*S[b0])  ^
     (3*S[b5]  | 2*S[b5]  | 1*S[b5]  | 1*S[b5])  ^
     (1*S[b10] | 3*S[b10] | 2*S[b10] | 1*S[b10]) ^
     (1*S[b15] | 1*S[b15] | 3*S[b15] | 2*S[b15])

R1 = (2*S[b4]  | 1*S[b4]  | 1*S[b4]  | 3*S[b4])  ^
     (3*S[b9]  | 2*S[b9]  | 1*S[b9]  | 1*S[b9])  ^
     (1*S[b14] | 3*S[b14] | 2*S[b14] | 1*S[b14]) ^
     (1*S[b3]  | 1*S[b3]  | 3*S[b3]  | 2*S[b3])

R2 = (2*S[b8]  | 1*S[b8]  | 1*S[b8]  | 3*S[b8])  ^
     (3*S[b13] | 2*S[b13] | 1*S[b13] | 1*S[b13]) ^
     (1*S[b2]  | 3*S[b2]  | 2*S[b2]  | 1*S[b2])  ^
     (1*S[b7]  | 1*S[b7]  | 3*S[b7]  | 2*S[b7])

R3 = (2*S[b12] | 1*S[b12] | 1*S[b12] | 3*S[b12]) ^
     (3*S[b1]  | 2*S[b1]  | 1*S[b1]  | 1*S[b1])  ^
     (1*S[b6]  | 3*S[b6]  | 2*S[b6]  | 1*S[b6])  ^
     (1*S[b11] | 1*S[b11] | 3*S[b11] | 2*S[b11])
```

It does not ends here…

- So finally

```
R0 = te0[b0]  ^ te1[b5]  ^ te2[b10] ^ te3[b15]
R1 = te0[b4]  ^ te1[b9]  ^ te2[b14] ^ te3[b3]
R2 = te0[b8]  ^ te1[b13] ^ te2[b2]  ^ te3[b7]
R3 = te0[b12] ^ te1[b1]  ^ te2[b6]  ^ te3[b11]
```

It doe

- So finall

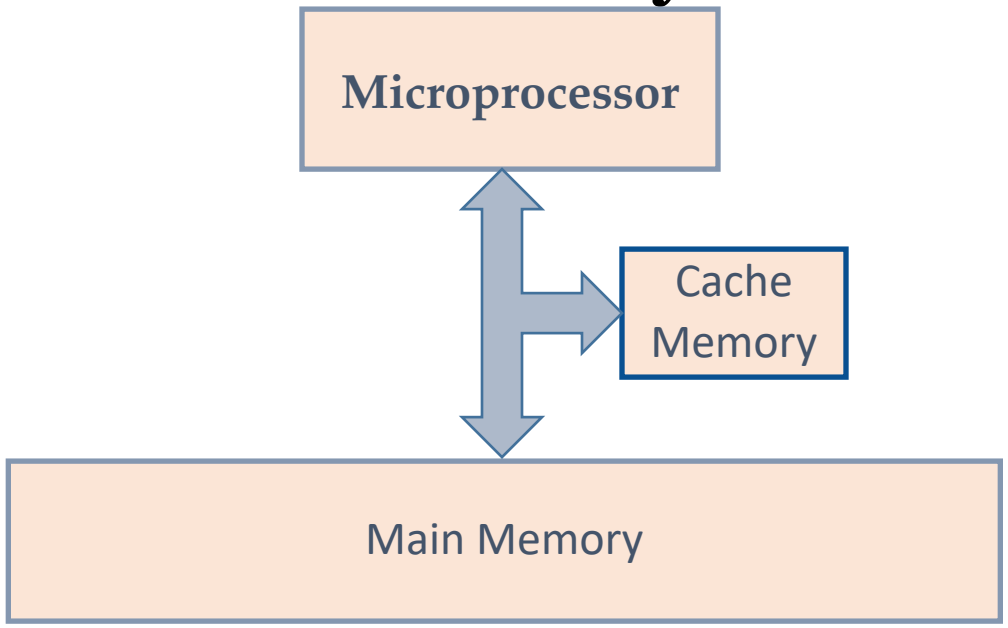
```
R0 = te0[b0]
R1 = te0[b4]
R2 = te0[b8]
R3 = te0[b12]
```

```
// Initialize the state, stored in s0, s1, s2 and s3
s0 := uint32(src[0])<<24 | uint32(src[1])<<16 | uint32(src[2])<<8 | uint32(src[3])
s1 := uint32(src[4])<<24 | uint32(src[5])<<16 | uint32(src[6])<<8 | uint32(src[7])
s2 := uint32(src[8])<<24 | uint32(src[9])<<16 | uint32(src[10])<<8 | uint32(src[11])
s3 := uint32(src[12])<<24 | uint32(src[13])<<16 | uint32(src[14])<<8 | uint32(src[15])

// Add the first round key to the state
s0 ^= xk[0]
s1 ^= xk[1]
s2 ^= xk[2]
s3 ^= xk[3]

for i:= 1; i < nr; i++ {
    // This performs SubBytes + ShiftRows + MixColumns + AddRoundKey
    tmp0 = te0[s0>>24] ^ te1[s1>>16&0xff] ^ te2[s2>>8&0xff] ^ te3[s3&0xff] ^ xk[4*i]
    tmp1 = te0[s1>>24] ^ te1[s2>>16&0xff] ^ te2[s3>>8&0xff] ^ te3[s0&0xff] ^ xk[4*i+1]
    tmp2 = te0[s2>>24] ^ te1[s3>>16&0xff] ^ te2[s0>>8&0xff] ^ te3[s1&0xff] ^ xk[4*i+2]
    tmp3 = te0[s3>>24] ^ te1[s0>>16&0xff] ^ te2[s1>>8&0xff] ^ te3[s2&0xff] ^ xk[4*i+3]

    s0, s1, s2, s3 = tmp0, tmp1, tmp2, tmp3
}
```

But As We Said…

- The tables are stored in cache memory of your system.
- AES accesses this table depending on the secret key values
- An adversary, who is able to measure the time for each encryption operation, and also using the same cache can do something so that it can recover the secret key!!!
- Such attacks are called cache timing attacks…

Attacks due to Memory Wall

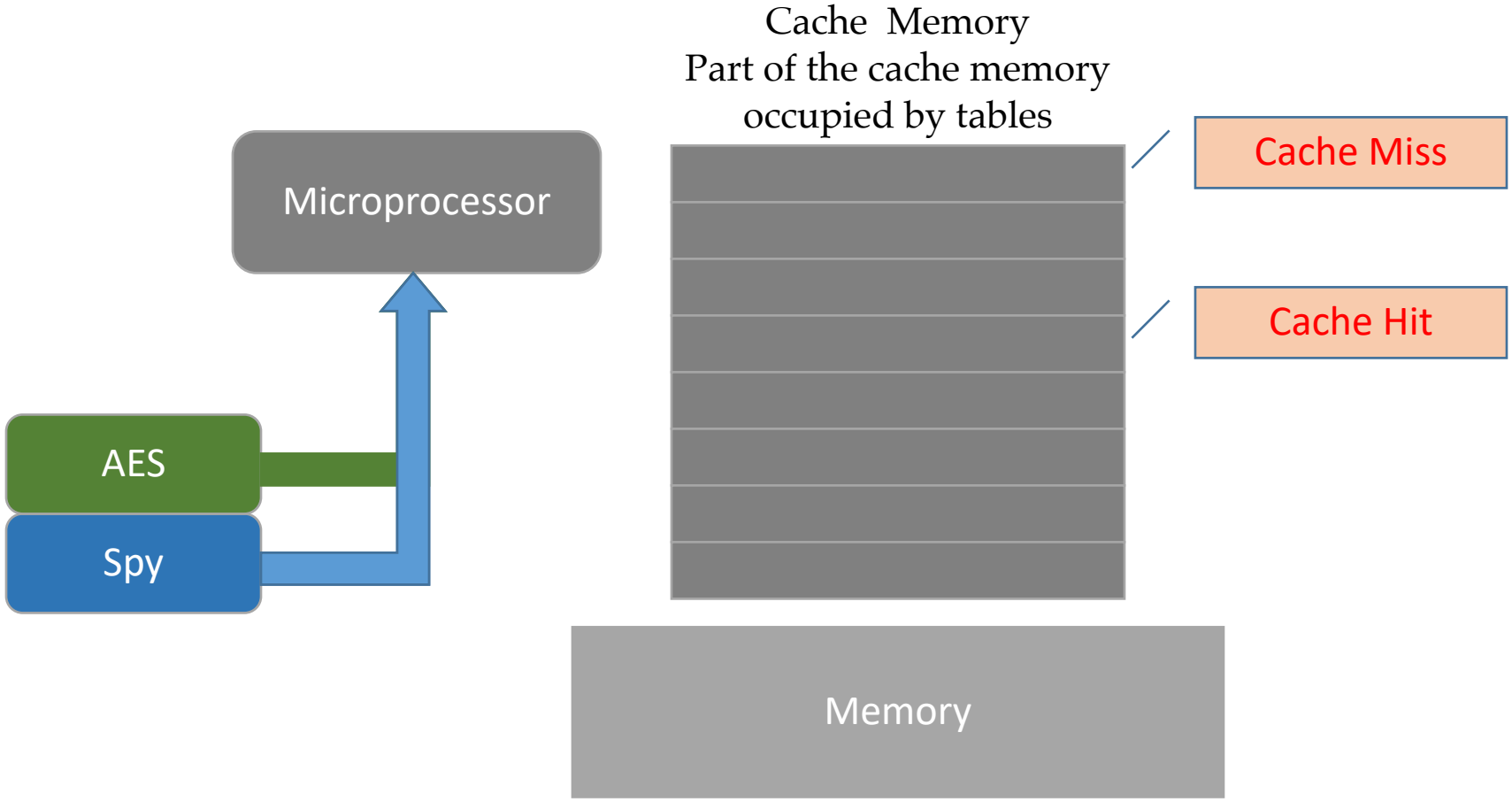


- If there is a *Cache Hit*
  - Access time is less
  - Power Consumption is less

- If there is a *Cache Miss*
  - Access time is more
  - Power Consumption is more

Timing Attacks due to Cache Memory

- Uses a spy program to determine cache behavior



Bitslicing

- Simply speaking, implement the software like hardware
- Results in constant-time crypto
- Idea: Let’s say you are running on a 32-bit machine.
  - 32-bit registers
  - Logical AND, OR, NOT, XOR.
  - Also consider your block cipher in terms of these gates.

Bitslicing: A Simple Example

- Let us consider the first equation only.
  - It can be written as follows:

```
and t1, x1, x4
and t2, t1, x2
and t3, t1, x3
xor t4, t2, t3
```

And so on…

$$y_1 = x_1x_2x_4 + x_1x_3x_4 + x_1 + x_2x_3x_4 + x_2x_3 + x_3 + x_4 + 1$$

$$y_2 = x_1x_2x_4 + x_1x_3x_4 + x_1x_3 + x_1x_4 + x_1 + x_2 + x_3x_4 + 1$$

$$y_3 = x_1x_2x_4 + x_1x_2 + x_1x_3x_4 + x_1x_3 + x_1 + x_2x_3x_4 + x_3$$

$$y_4 = x_1 + x_2x_3 + x_2 + x_4$$

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  - It can be written as follows:

```
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```

And so on…

$$y_1 = x_1x_2x_4 + x_1x_3x_4 + x_1 + x_2x_3x_4 + x_2x_3 + x_3 + x_4 + 1$$

$$y_2 = x_1x_2x_4 + x_1x_3x_4 + x_1x_3 + x_1x_4 + x_1 + x_2 + x_3x_4 + 1$$

$$y_3 = x_1x_2x_4 + x_1x_2 + x_1x_3x_4 + x_1x_3 + x_1 + x_2x_3x_4 + x_3$$

$$y_4 = x_1 + x_2x_3 + x_2 + x_4$$

- Now, each of t1, x1, x2,… are mapped to 32 bit registers; but actually they are processing 1-bit values
  - So, what to do?

Bitslicing: A Simple Example

- Let us consider the first equation only.
  - It can be written as follows:

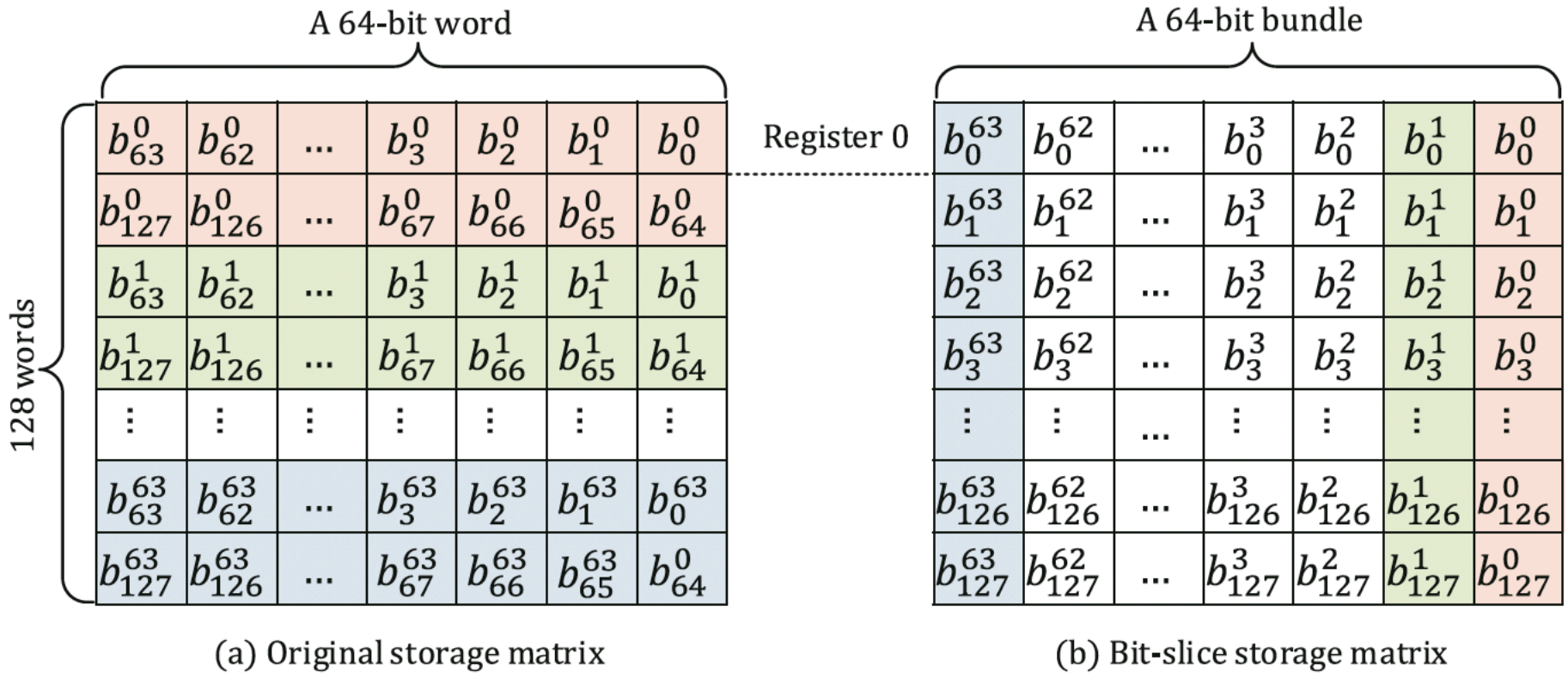
```
and t1, x1, x4
and t2, t1, x2
and t3, t1, x3
xor t4, t2, t3
```

And so on…

- Pack each register with independent values and process them using the same instruction.!!!
- Easiest case: you can encrypt 32 plaintext together

Bitslicing: A Simple Example

- Easiest case: you can encrypt 32 plaintext together
- You can parallelize S-Box computations for one plaintext

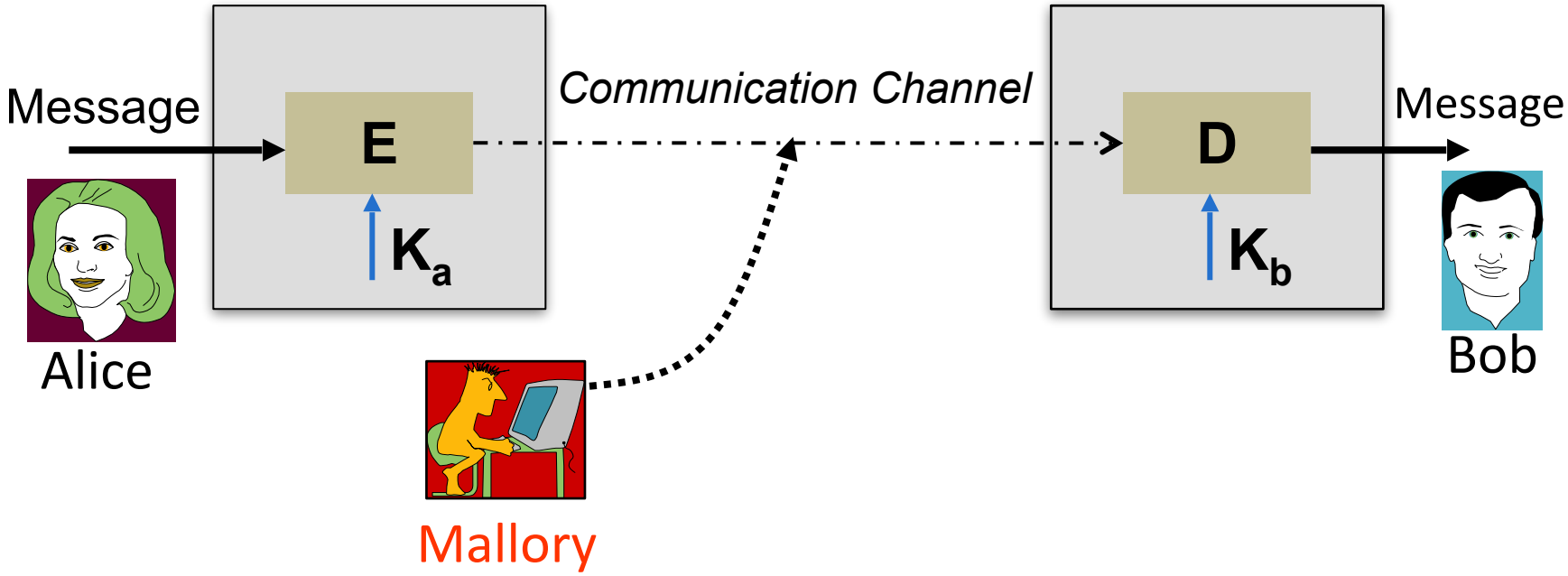


(a) Original storage matrix

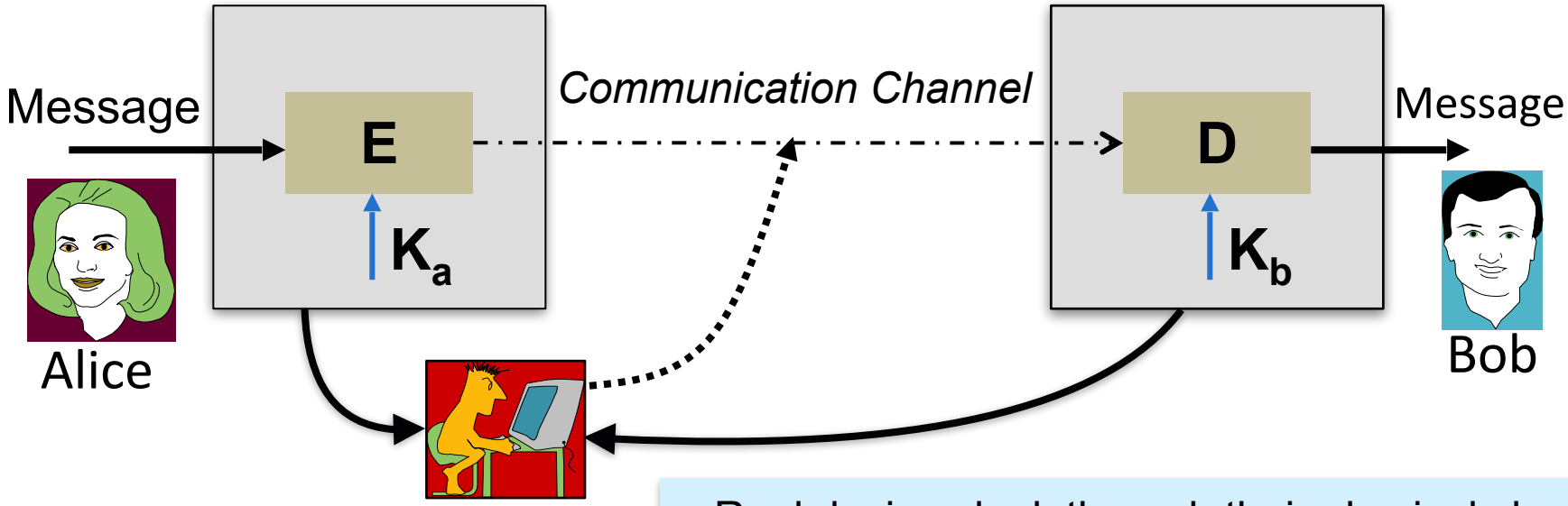
(b) Bit-slice storage matrix

Side Channel Attacks

Why do Cryptographers Need Engineers?



Cryptographic Security: Real World



Strong cryptographic algorithms are only the beginning

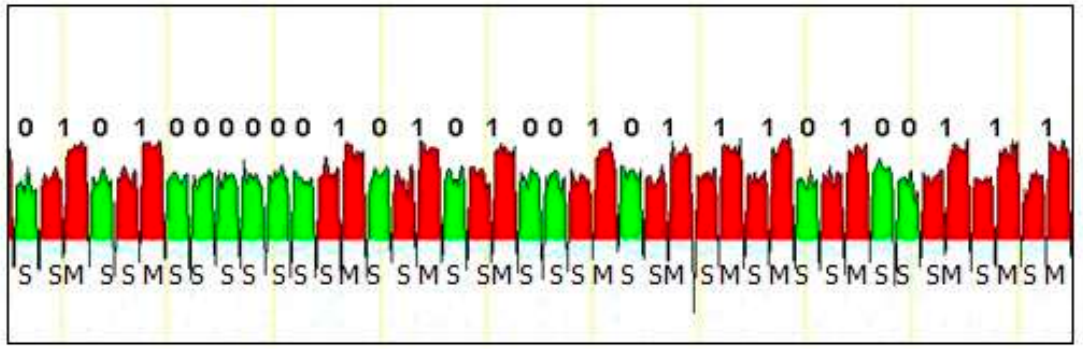
Real devices leak through their physical characteristics

- Timing
- Power consumption
- Electromagnetic Radiation
- Sound
- Faults

Analysis and mitigation of physical attacks are cryptographic as well as engineering problems

Side-Channel Attacks (SCA)

- The physical channels are correlated with the information being processed
- Fundamental cause: power consumption is correlated with switching of CMOS transistors (0->1, 1->0)
    - Typically it is assumed that power consumption is correlated with the Hamming Weight/ Distance.
- If some internal state is exposed, the secret key can be recovered in seconds.



Source: Internet

Encryption

Plaintext	$M < n$
Ciphertext	$C = M^e \pmod n$

Decryption

Ciphertext	C
Plaintext	$M = C^d \pmod n$

Square and Multiply Algorithm

Goal: Compute $a^e \pmod n$

1. convert e to binary: $k_s k_{s-1} \ldots k_1 k_0$
2. b = 1;
3. for (i=s; i>=0; i--)
4. { b = b*b (mod n);
5. if (k_i == 1)
6. b = b * a (mod n)
7. }
8. return b;

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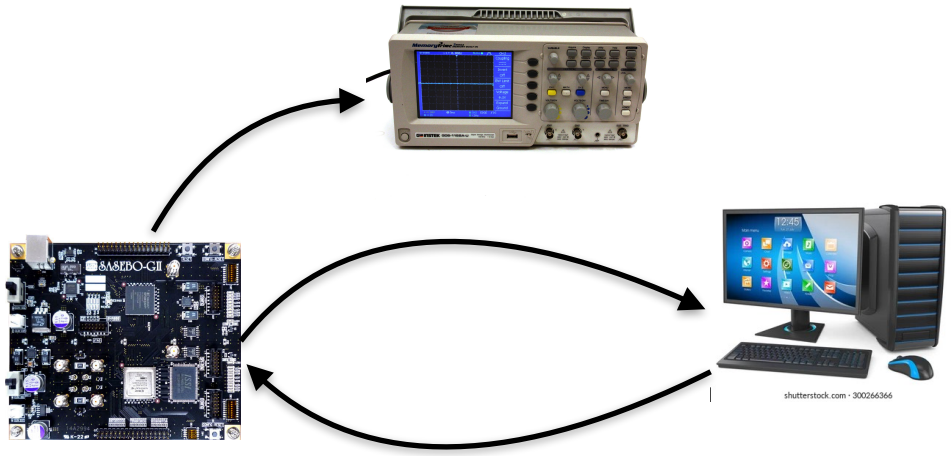

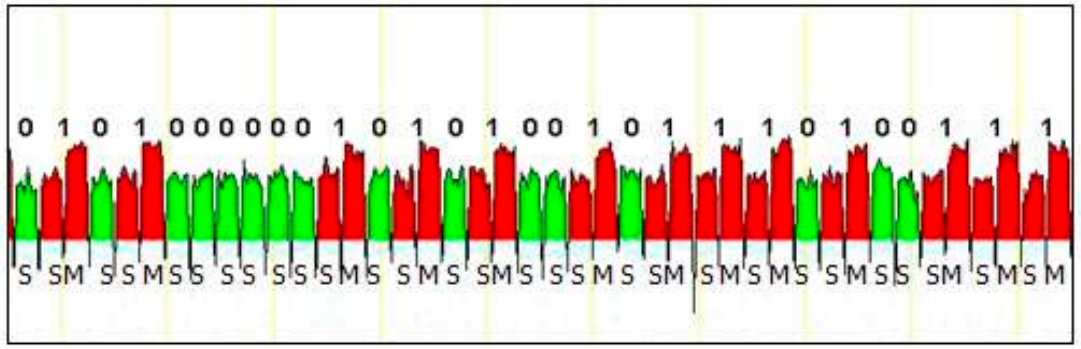

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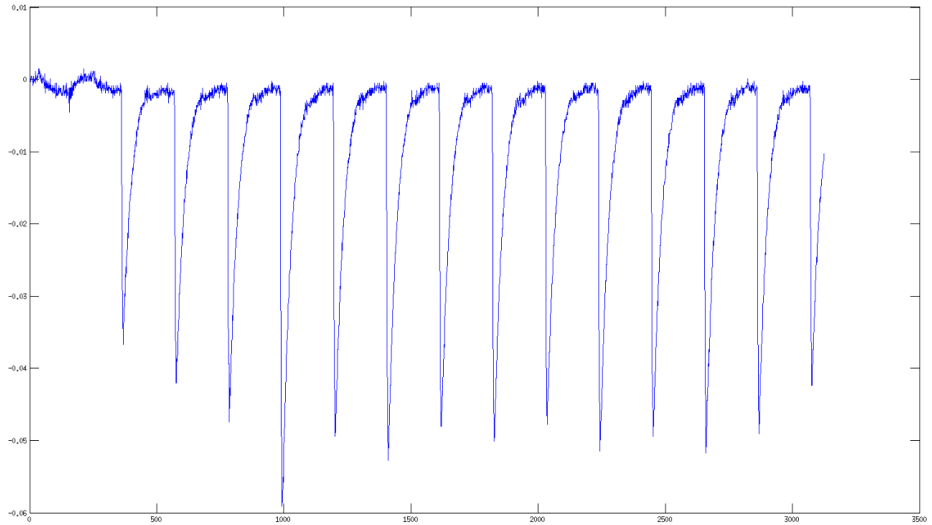

Source: Testbed for Side Channel analysis and security evaluation

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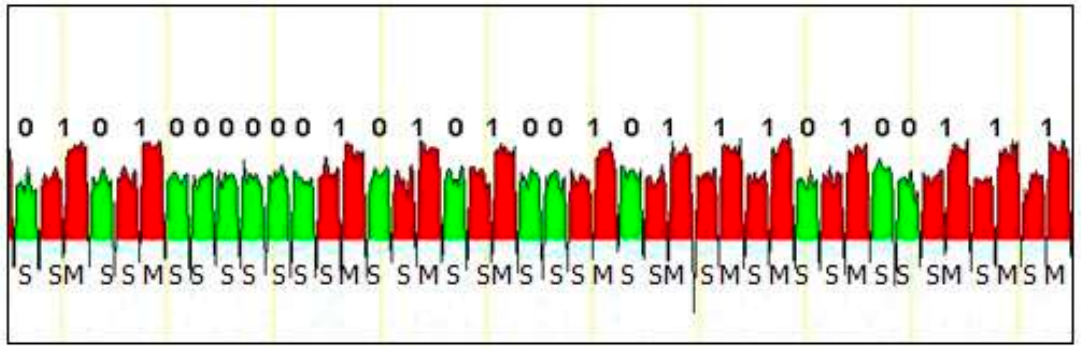

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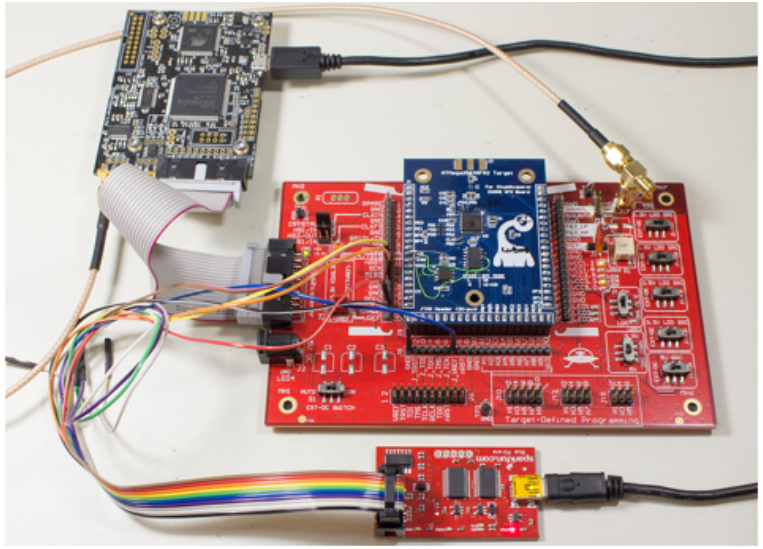

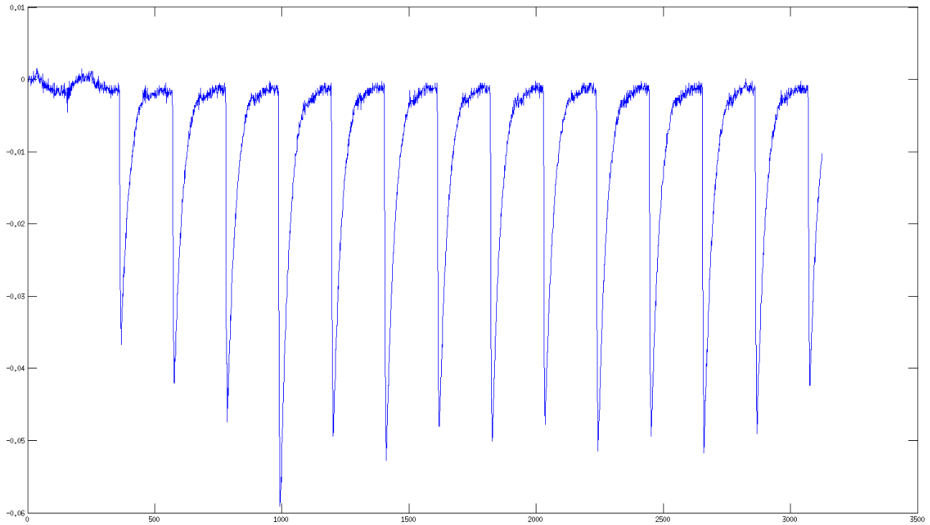

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Side-Channel Vs. Classical Cryptanalysis

- Cryptanalysis: Purely mathematical
  - Take example of AES
  - Cryptanalysis means, you only have access to plaintext, ciphertext — a lot of them
  - You have to
    - Find the key
    - Or, at least, show that it is distinguishable from uniform randomness

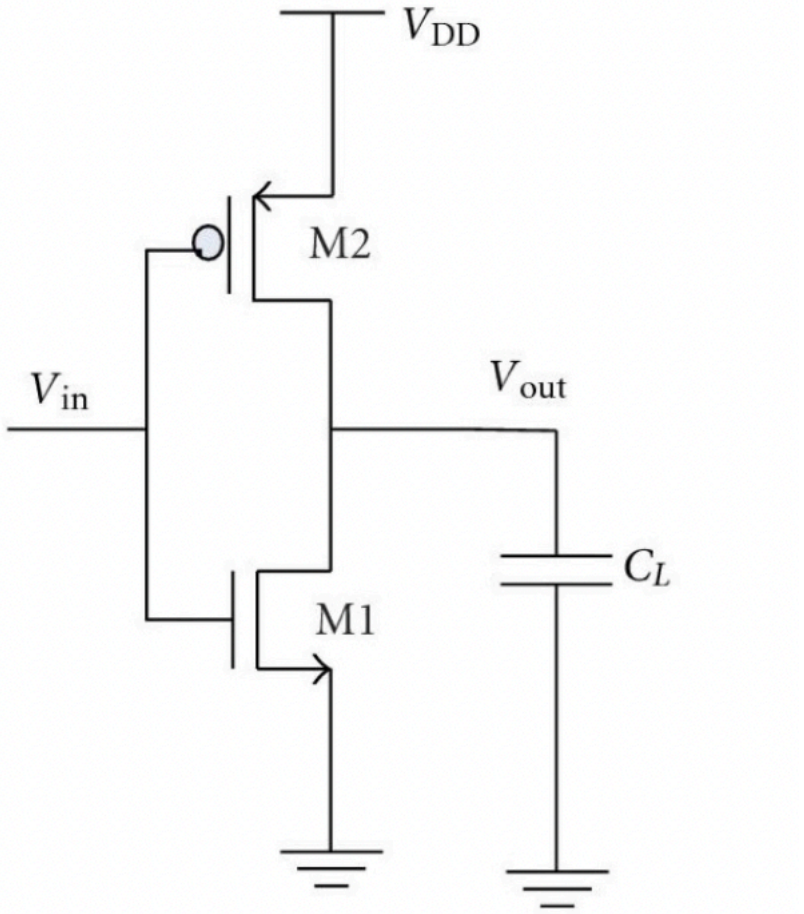
Side-Channel Vs. Classical Cryptanalysis

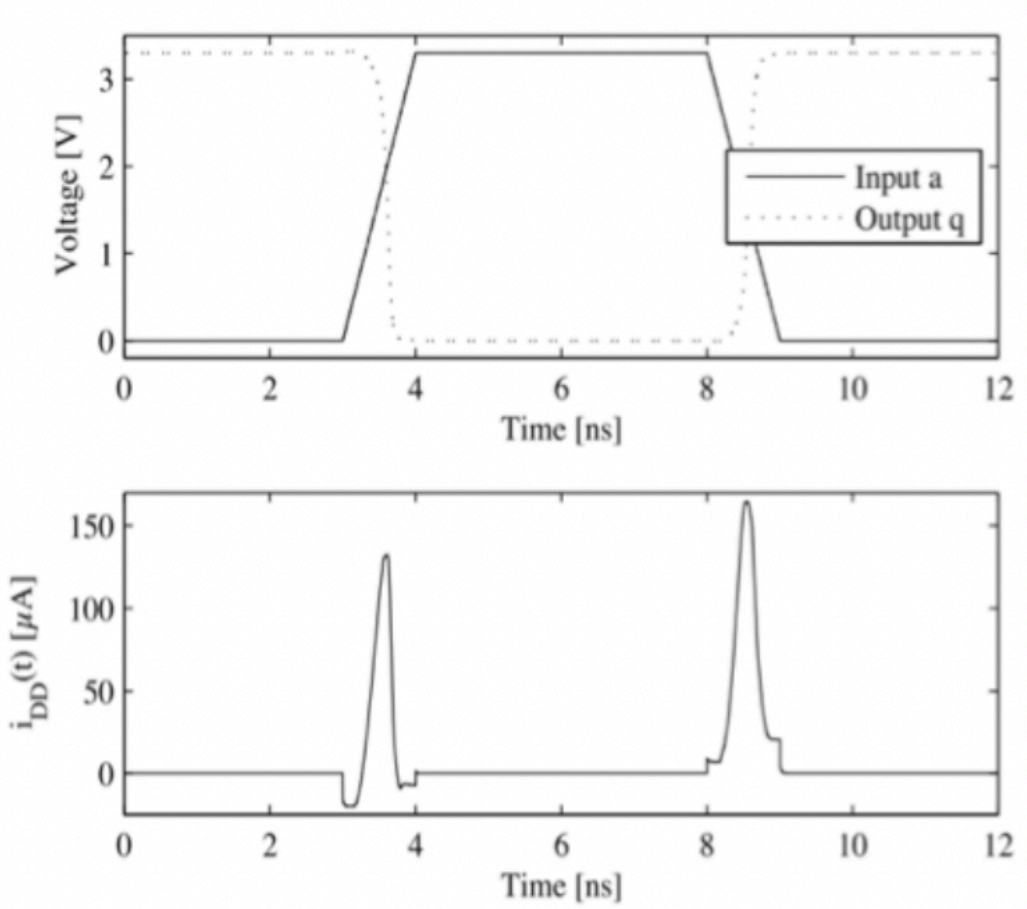
- Cryptanalysis: Purely mathematical
  - Take example of RSA/ECC/PQC
  - Cryptanalysis means, you only have access to plaintext, ciphertext — a lot of them
  - You have to
    - Find the key
      - Maybe you need to solve the underlying hard problem in some (mathematical) way.

Side-Channel Vs. Classical Cryptanalysis

- Side-Channel Cryptanalysis: Mathematics + Physics + Statistics
  - The goal is mostly to recover key
    - But also signature forgery, confidentiality breach
  - Ranges beyond crypto…
    - Kernel information extraction
    - Unprivileged access
    - Neural network reverse engineering

The Root Cause





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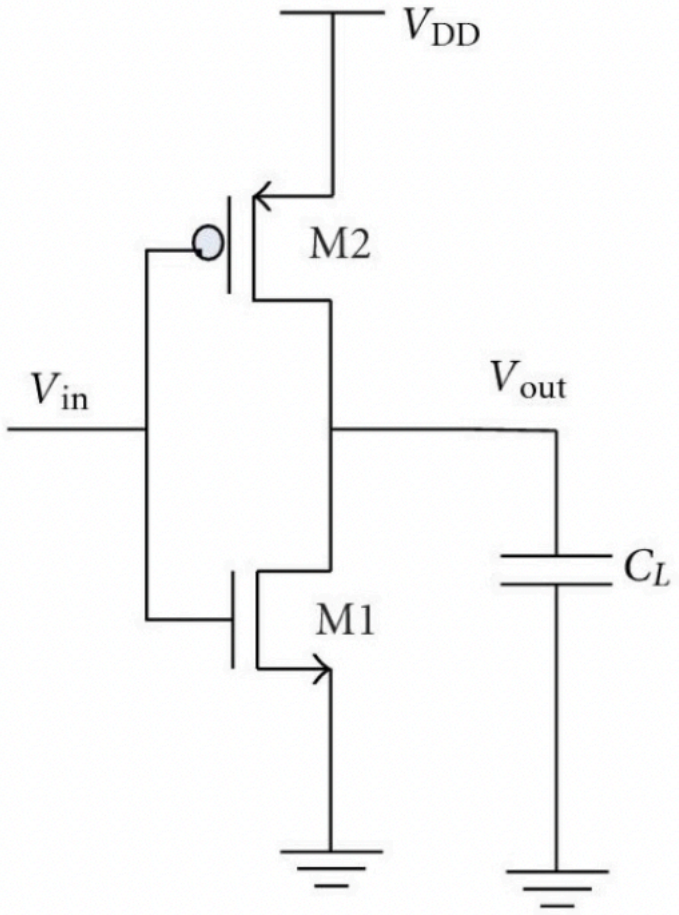
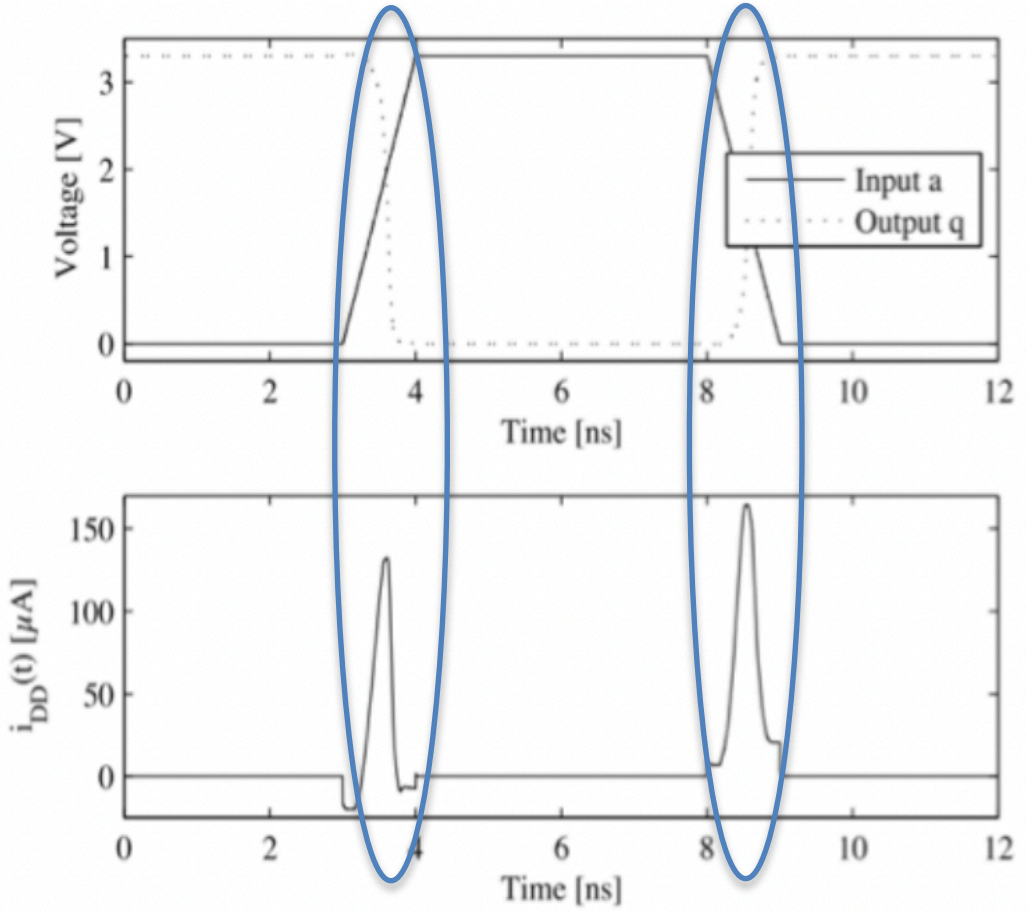
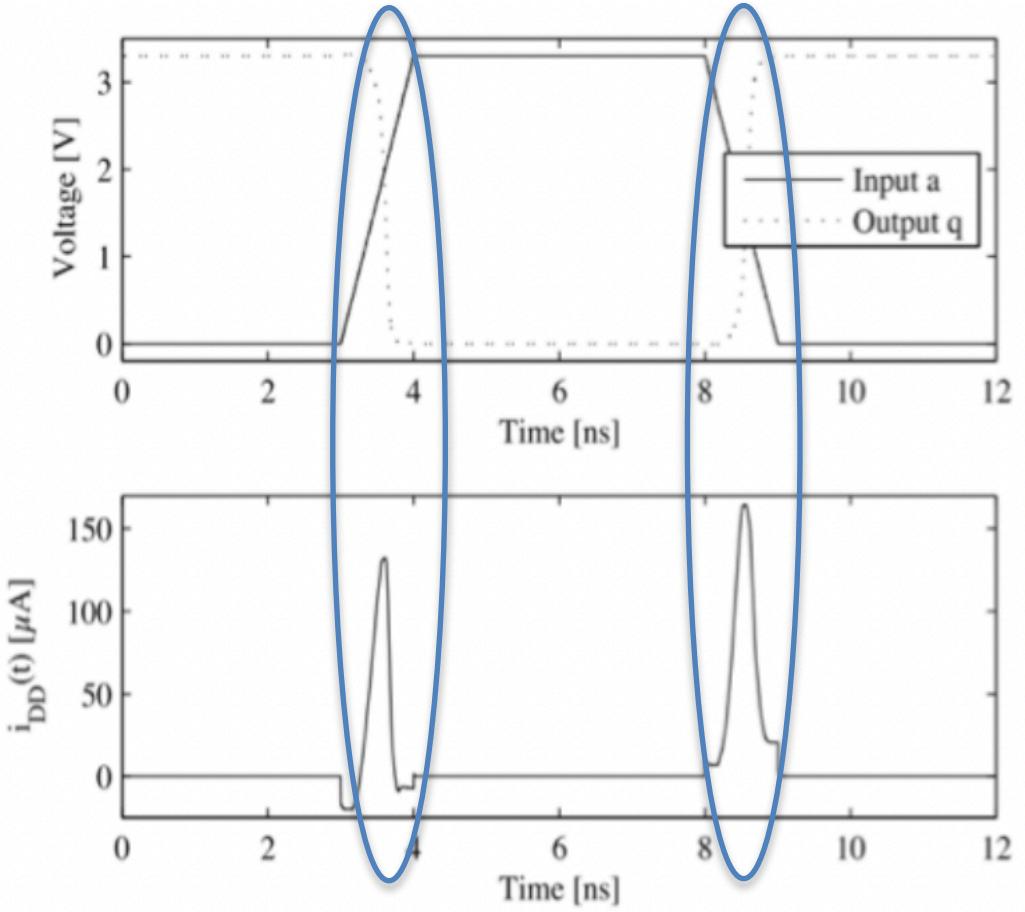




The Root Cause

What is exploited?

- The state change of a gate is proportional to the power dissipated.
- Think about a circuit with millions of gates.
- **How to measure**
  - Power dissipation can be measured by putting a resistor in series with Vdd or Vss and the true source/ground.
  - Roughly, 1 Ohm resistors work well for many microcontrollers, but it is highly target dependent
  - We actually measure current.
    - Differential probes.
  - **The best approach is to use a near-field H-probe an measure EM signal**
    - Less noisy than global power measurement



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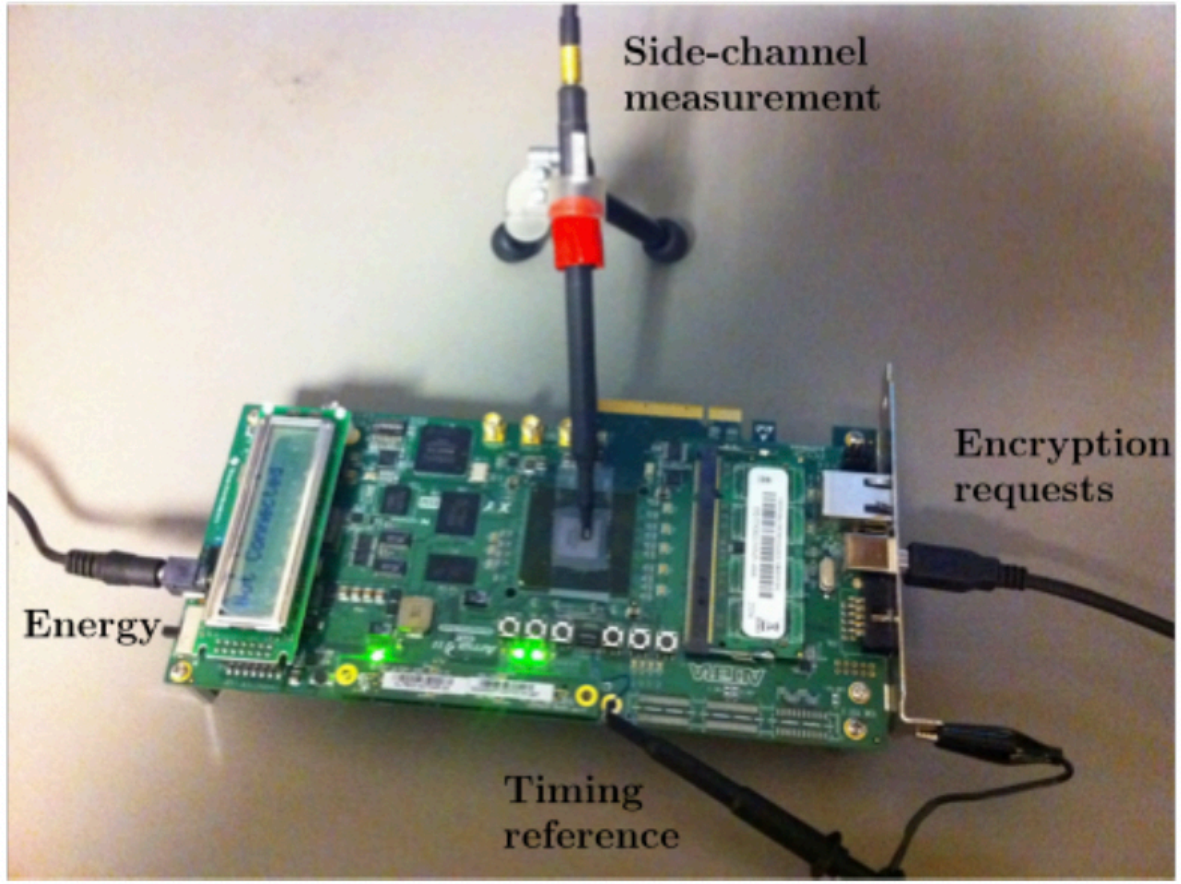
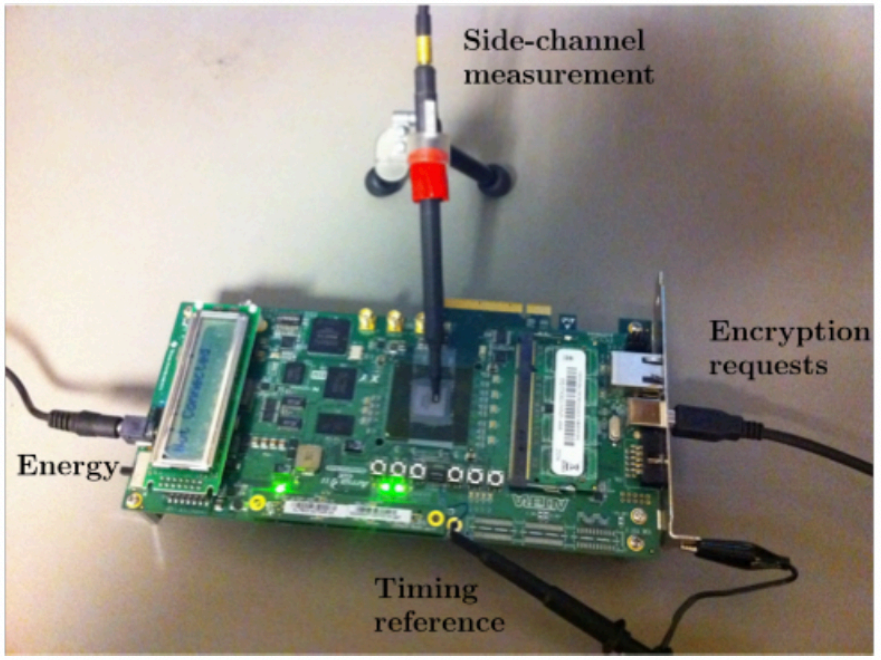


What to “Measure”?

The Crypto Running on a Microcontroller/ FPGA/ASIC

- End of the day everything is CMOS!!!
- Since power consumption is proportional to the switching activity, *so we can get some idea about the internal computation of the crypto*
  - **The crypto is no more black box**
- In this talk we will be specifically focusing on symmetric key algorithms
  - **AES**
- **What do we mean by attacking AES?**
  - **Finding out it’s secret key**



Looking Inside AES

State

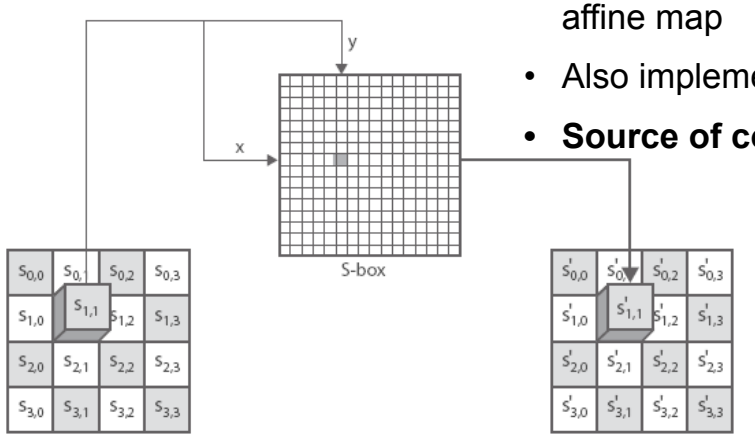
1 byte

s00	s01	s02	s03
s10	s11	s12	s13
s20	s21	s22	s23
s20	s31	s32	s33

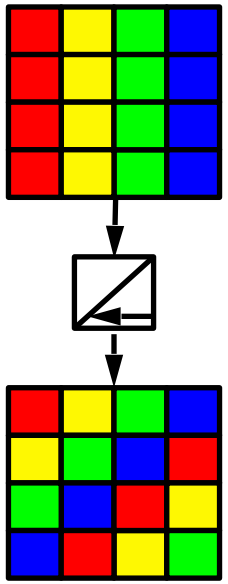
k00	k01	k02	k03
k10	k11	k12	k13
k20	k21	k22	k23
k20	k31	k32	k33

1. SubBytes

- Nonlinear Boolean Function
- Finite field inversion followed by affine map
- Also implemented as a table
- **Source of confusion**

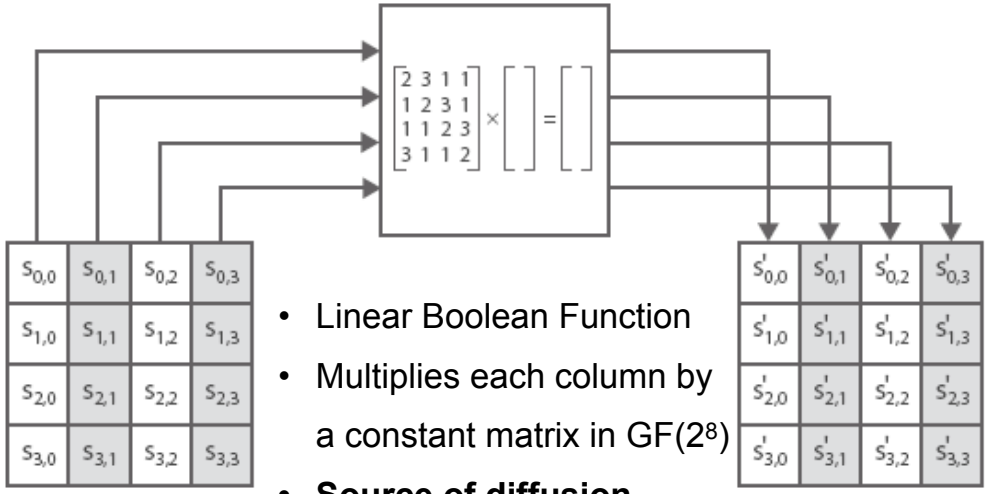


2. ShiftRows



- Linear Boolean Function
- Left circular shift of rows
- **Source of diffusion**

3. MixColumns



- Linear Boolean Function
- Multiplies each column by a constant matrix in GF(2^8)
- **Source of diffusion**

4. AddRoundKey

s00	s01	s02	s03
s10	s11	s12	s13
s20	s21	s22	s23
s20	s31	s32	s33

$\oplus$

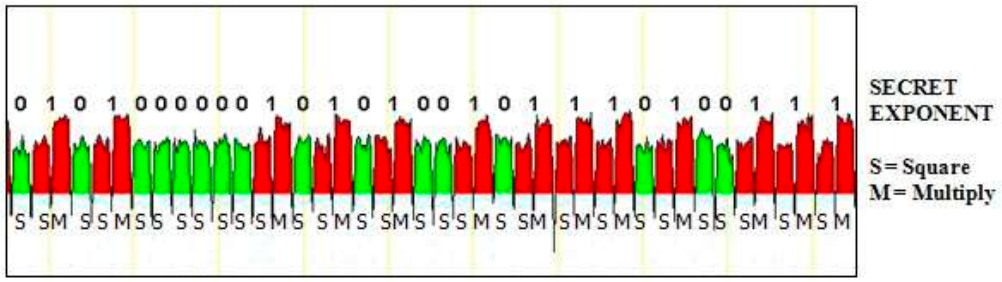
k00	k01	k02	k03
k10	k11	k12	k13
k20	k21	k22	k23
k20	k31	k32	k33

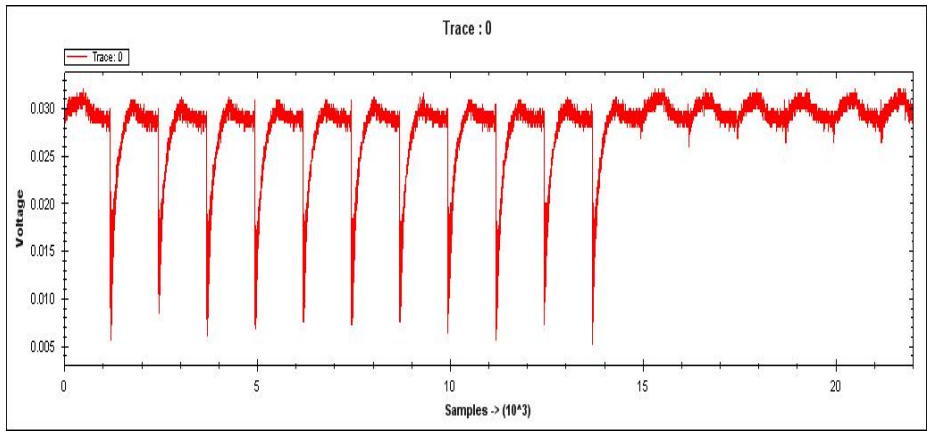
- Linear Boolean Function
- XOR the state with a round key

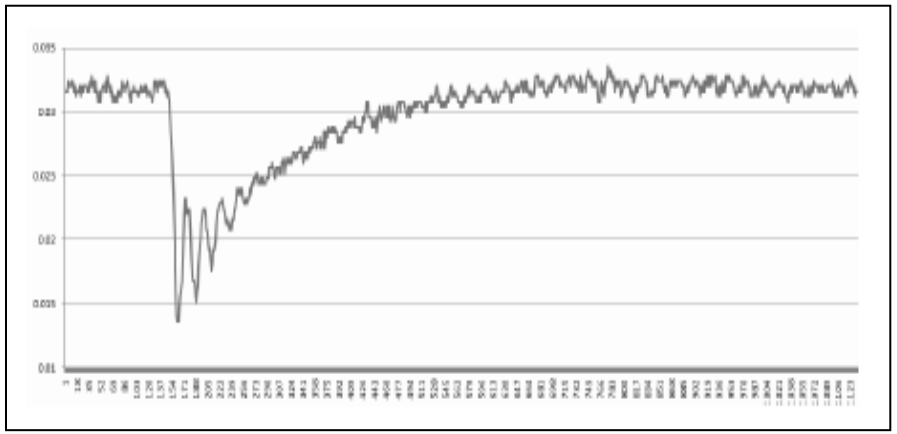
Through the Looking Glass

There can be two kinds of power analysis attacks:

- **Simple power analysis (SPA)**: Exploits the operation dependence of power consumption
  - Remember the RSA example from the beginning…
- **Differential Power Analysis (DPA):** Exploits the data dependence of power consumption
  - We well see now for AES
  - **Fact: there is no secret dependent operation in AES, everything is uniform.**



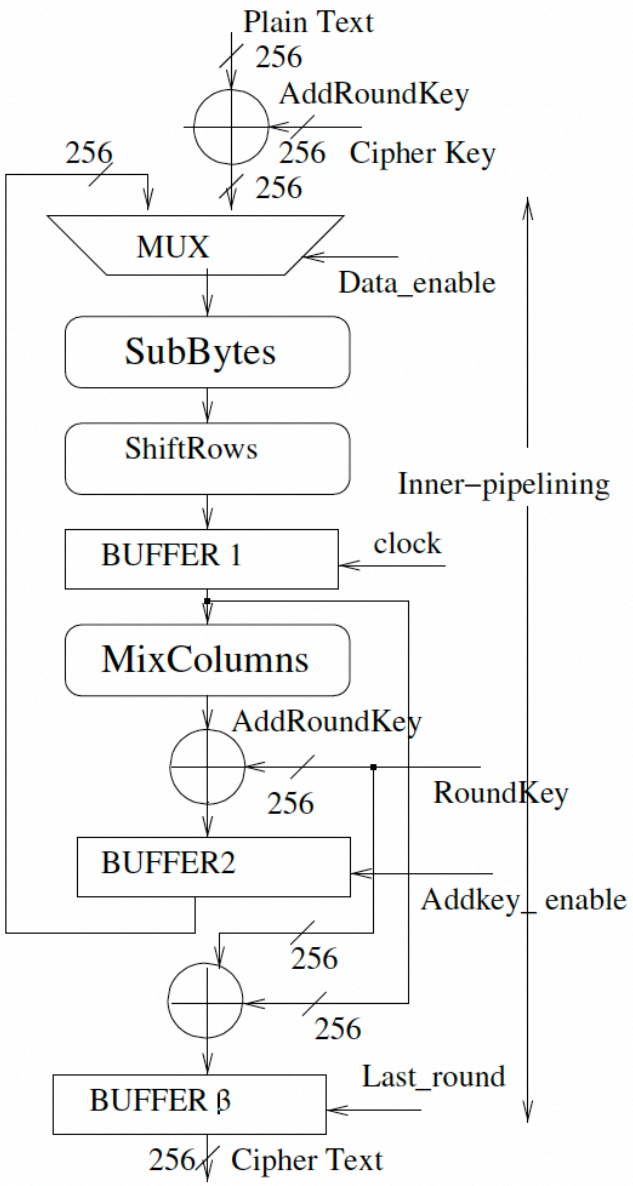




Differential Power Analysis

- **Power Trace**: A set of power consumptions across a cryptographic process
    - 1 millisecond operation sampled at 5MHz yield a trace with 5000 points.
- **Leakage Model**: Hypothetical model relating the leakage with the internal states of the target algorithm.
    - For AES the internal state is a 128-bit value.
    - **Hamming Weight Model:** The power consumption is proportional to the Hamming weight (count of 1's) of the state.
    - **Hamming Distance Model**: The power consumption is proportional to the Hamming distance between the state in two consecutive clock cycles. — FPGA/ASICs
    - More complex models are possible…
    - Used to simulate leakage and also in some attacks.

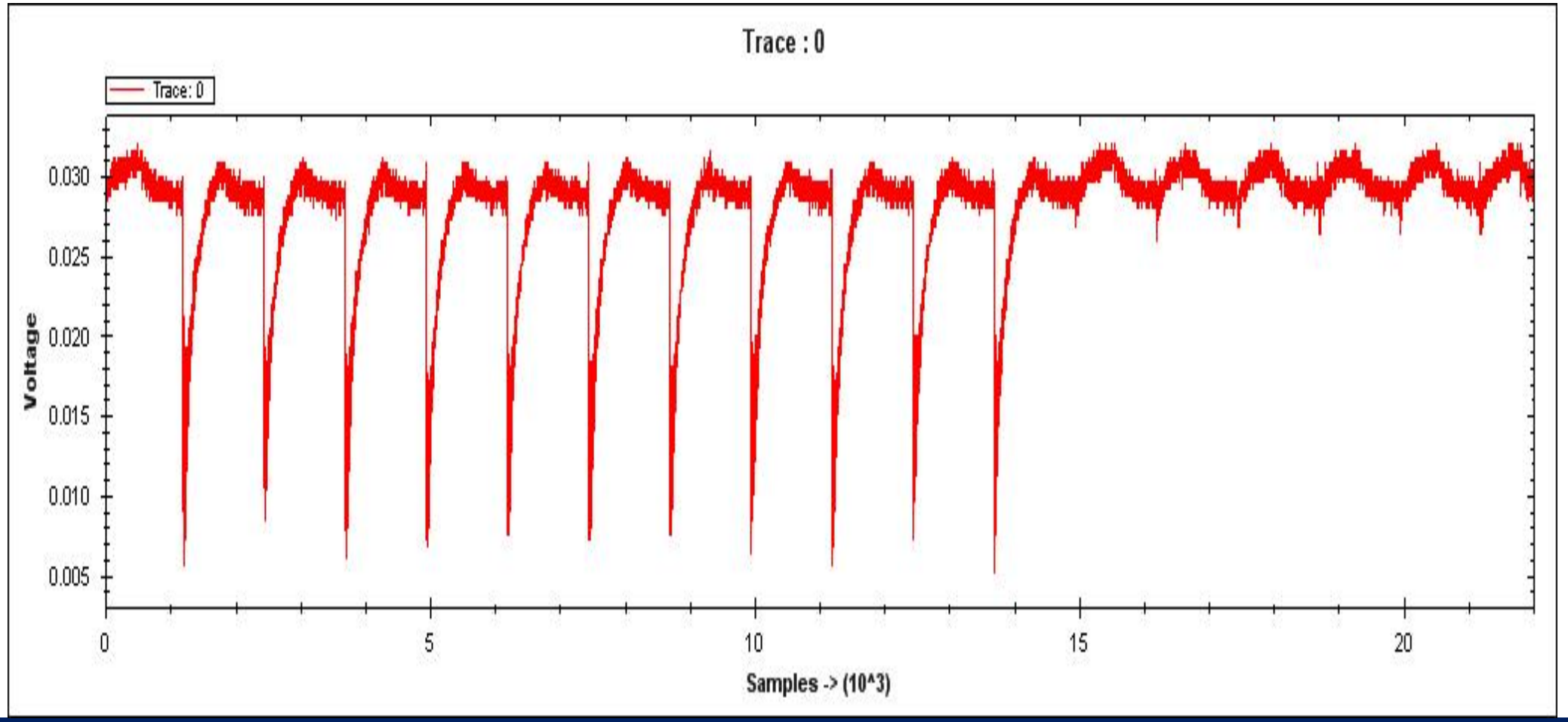
Differential Power Analysis



- Common hardware implementation of a block cipher
- We consider the state of the circuit at time instance t — you can consider it as one time point in the x-axis of the trace.
- Let this state be v_t
- Hamming weight is number of 1's in v_t.
- Hamming distance is HW($v_t \oplus v_{t-1}$)..
  - Why?

Differential Power Analysis

- **Power Trace**: A set of power consumptions across a cryptographic process
  - 1 millisecond operation sampled at 5MHz yield a trace with 5000 points.
-



Differential Power Analysis: The Idea

s	HW(s)	Target bit (LSB)
0000	0	0
0001	1	1
0010	1	0
0011	2	1
0100	1	0
0101	2	1
0110	2	0
0111	3	1
1000	1	0
1001	2	1
1010	2	0
1011	3	1
1100	2	0
1101	3	1
1110	3	0
1111	4	1

- Assume power leakage follows Hamming Weight.
- Divide the HW(s) into two bins:
  - 0 bin: when LSB is 0
  - 1 bin: when LSB is 1

Differential Power Analysis: The Idea

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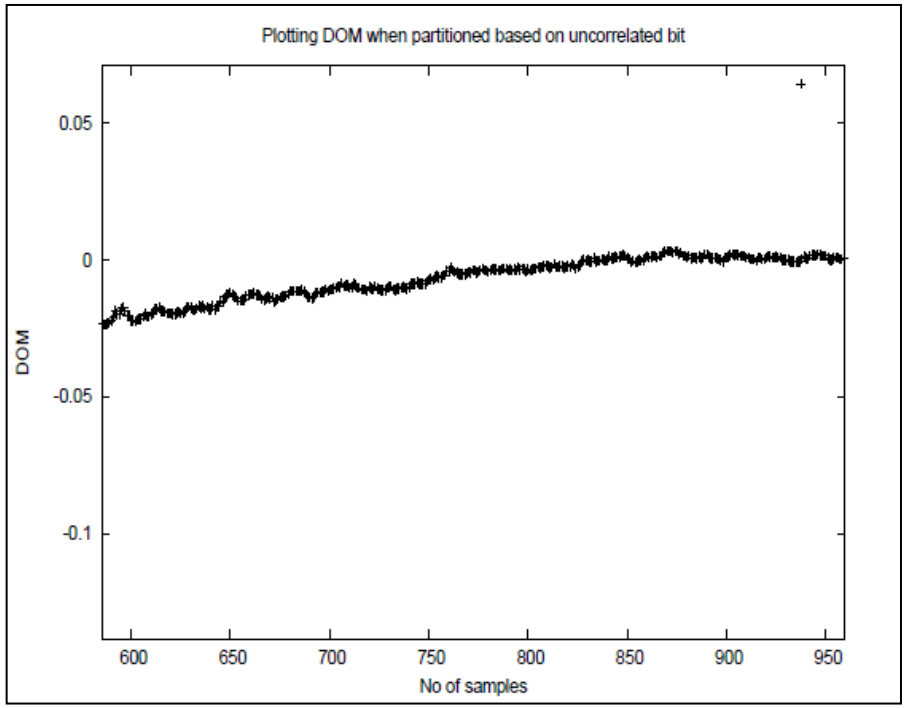
- Assume power leakage follows Hamming Weight.
- Divide the HW(s) into two bins:
  - 0 bin: when LSB is 0
  - 1 bin: when LSB is 1
- Difference-of-Mean (DoM)=20/8-12/8=1

When the Partitioning is Random



- Parititioning done by bits simulated using *rand* function in C.
- Observe the DoM is close to 0, as expected!
- **Note**: Instead of computing the difference, we can use some statistical hypothesis test, such as t-test.
- The hypothesis will be — whether the two trace distributions are the same or different
- For a random uncorrelated bit, the two distributions are the same.

- Moral of the story: If the bin partitioning is based on a bit from the actual state, there will be significant difference in the mean values of the bins. This is because, the traces are data dependent.

Attacking AES

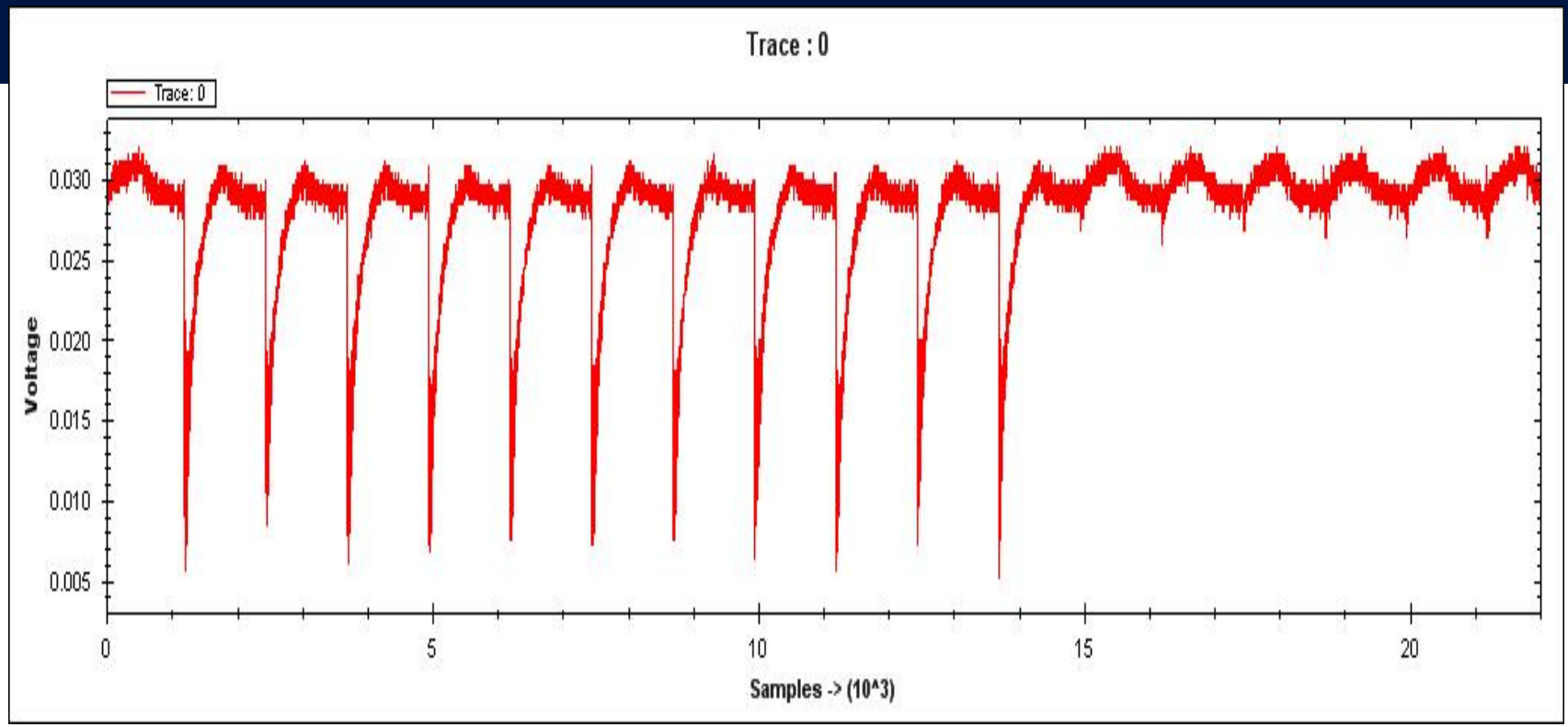
A very very important point

- We haven't seen AES key schedule in detail — but it is has somewhat similar operations as the rounds — has SBoxes, shifts XORs etc.
- **Important**: key schedule is invertible
  - That is, if you recover one round key , you can recover all, and the master key

Attacking AES

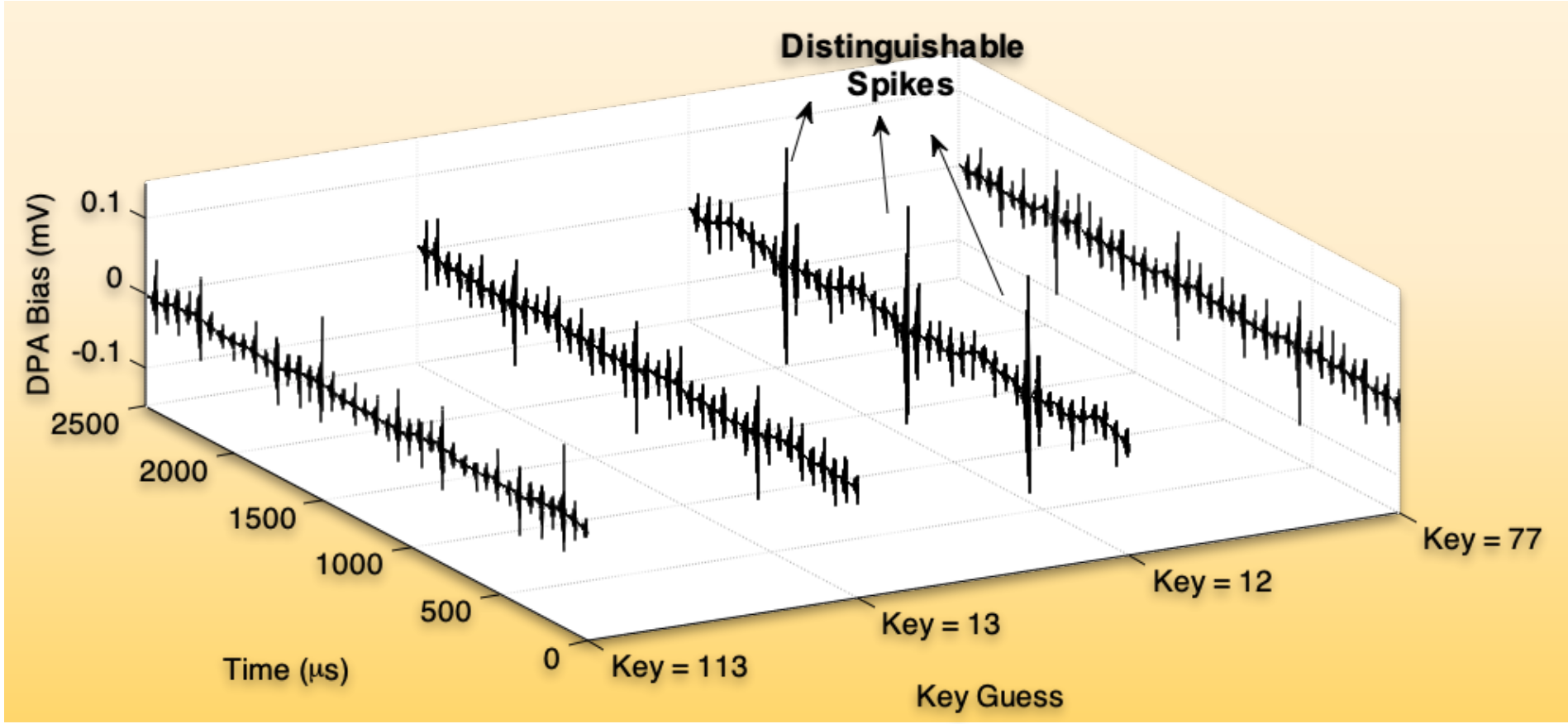
1. **Measurement**:
   - Make power consumption measurement of about 1000 AES operations, 100000 data points/trace,
   - Save (Ciphertext$_i$, trace_i)
2. **Attack**:
   - Target an S-Box in the last round (say the *j*-th S-Box)
     A. Guess a key for an S-box of last round (8 bit key, so total 256 guesses possible)
     B. Partially decrypt one byte of each ciphertext with the guessed key till the input of the last round S-Box. That is compute: $S_j = S^{-1}(C_j \oplus k_j^g)$
     C. Divide the traces into 2 groups based on the LSB of S_j
     D. Calculate the **average trace** of each group
     E. Calculate the difference of two **average traces**
     F. **Correct key guess → spikes in the differential curve**
   - Repeat A-F for other S-boxes

Attacking AES



CT[0]	T[0][0]	T[0][1]	…	T[0][m]
CT[1]	T[1][0]	T[1][1]	…	T[1][m]
CT[2]	T[2][0]	T[2][1]	…	T[2][m]
…	…	…	…	…
CT[n]	T[n][0]	T[n][1]	…	T[n][m]

Attacking AES



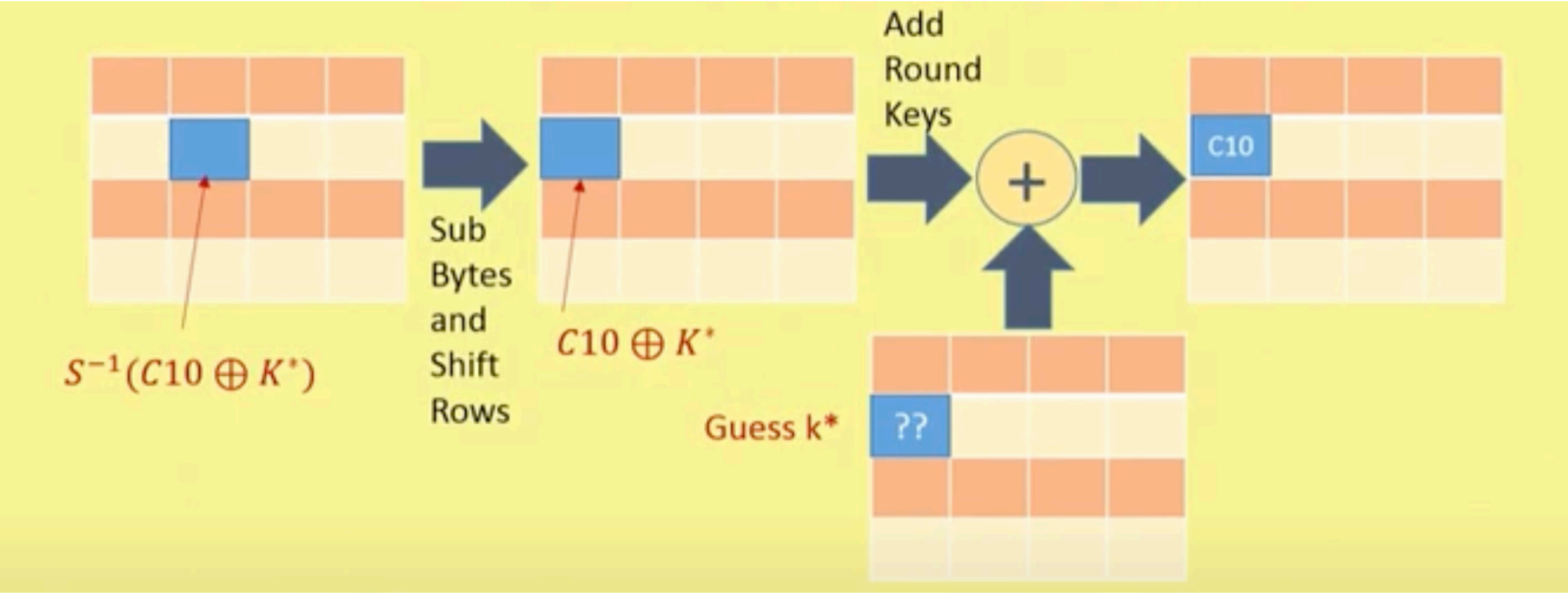
SBOX – 11 BIT – 8 TRACE COUNT = 15,000, FPGA implementation

Attacking AES

- **DPA selection function**: D(C,b,k^g) is defined as computing the value of the
  - b^{th} output bit, depending upon
    - C: Ciphertext
    - k^g is the guessed key for the S-Box
- In the attack, $D = S^{-1}(C_j \oplus k_j^g)$
- If k^g is a wrong guess then *b* is correctly evaluated only for half of the ciphertexts (randomly).
  - **Thus for large number of points, the difference between average traces is close to 0**
  - **In other words, distribution of both the bins will be the same**
- But if k^g is a correct guess, then *b* is correctly evaluated for all the ciphertexts.

Principle: If K_s is wrongly guessed, D behaves like a random guess. Thus for a large number of sample points, Δ[1..k] tends to zero. But if its correct, the differential will be non-zero and show spikes when D is correlated with the value being processed.

Attacking AES



$$D(C10, b = 0, K10) = S^{-1}(C10 \oplus K^*)|_{b=0}$$

Attacking AES

- **Differential Trace**: It is a *m* sample trace denoted as Δ_D, where,

$$\Delta_D[j] = \frac{\sum_{i=0}^{n-1} D(C_i, b, K_g)T[i][j]}{\sum_{i=0}^{n-1} D(C_i, b, K_g)} - \frac{\sum_{i=0}^{n-1} (1 - D(C_i, b, K_g))T[i][j]}{\sum_{i=0}^{n-1} (1 - D(C_i, b, K_g))}$$

- **Note**: C_i is a particular byte of the *i*-th ciphertext.

Attacking AES

- **Why does the attack work?**
  - It's not only the data dependency. But it also depends on the mathematics of AES
  - **DPA selection function**: D(C,b,k^g) is defined as computing the value of the
    - b^{th} output bit, depending upon
      - C: Ciphertext
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    - **In other words, distribution of both the bins will be the same**
  - But if k^g is a correct guess, then *b* is correctly evaluated for all the ciphertexts.
  - **Note**: The non-linearity of the S-Boxes play an important role here.

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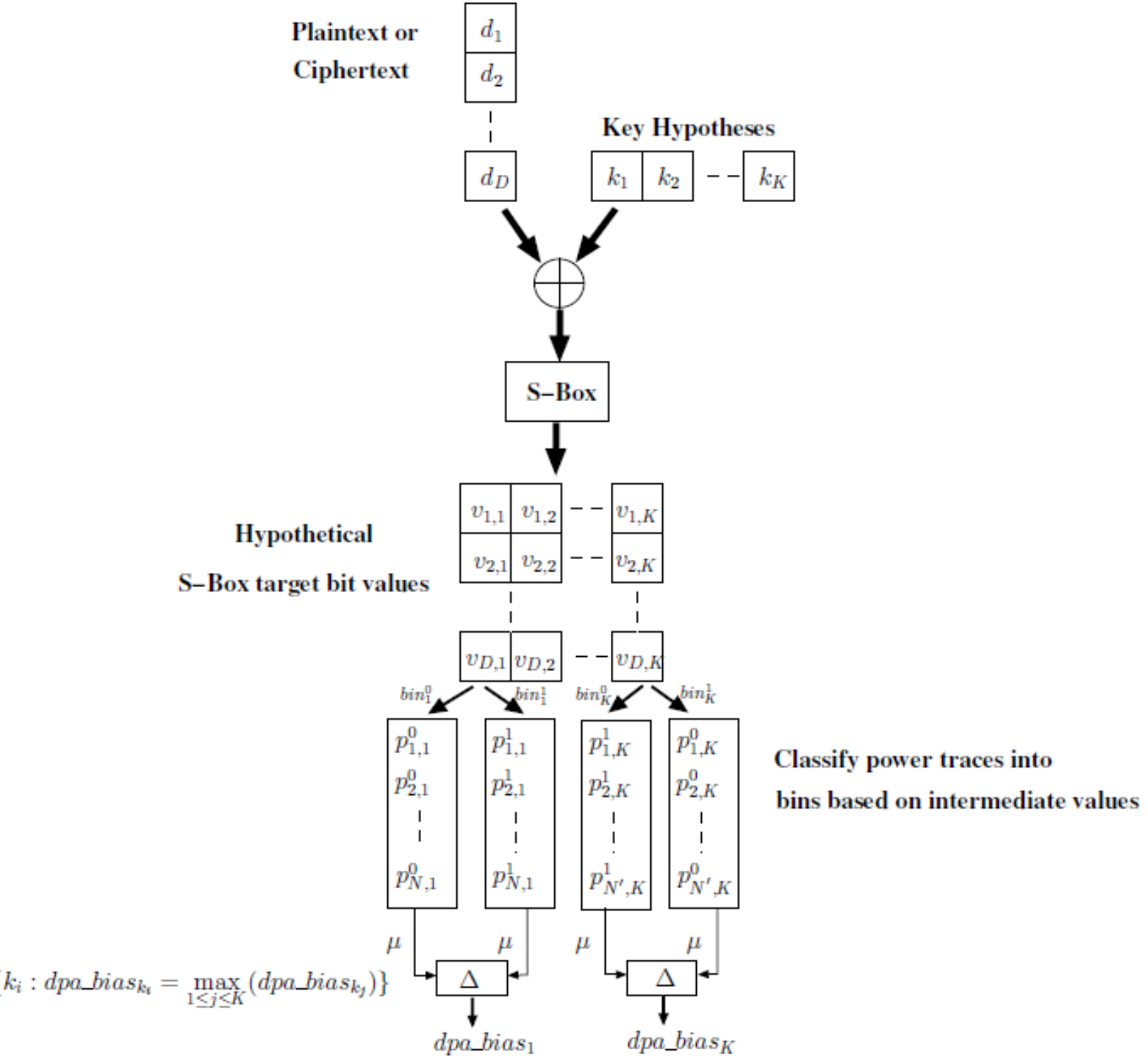


$$Correct\ key = \{k_i : dpa\_bias_{k_i} = \max_{1 \le j \le K}(dpa\_bias_{k_j})\}$$

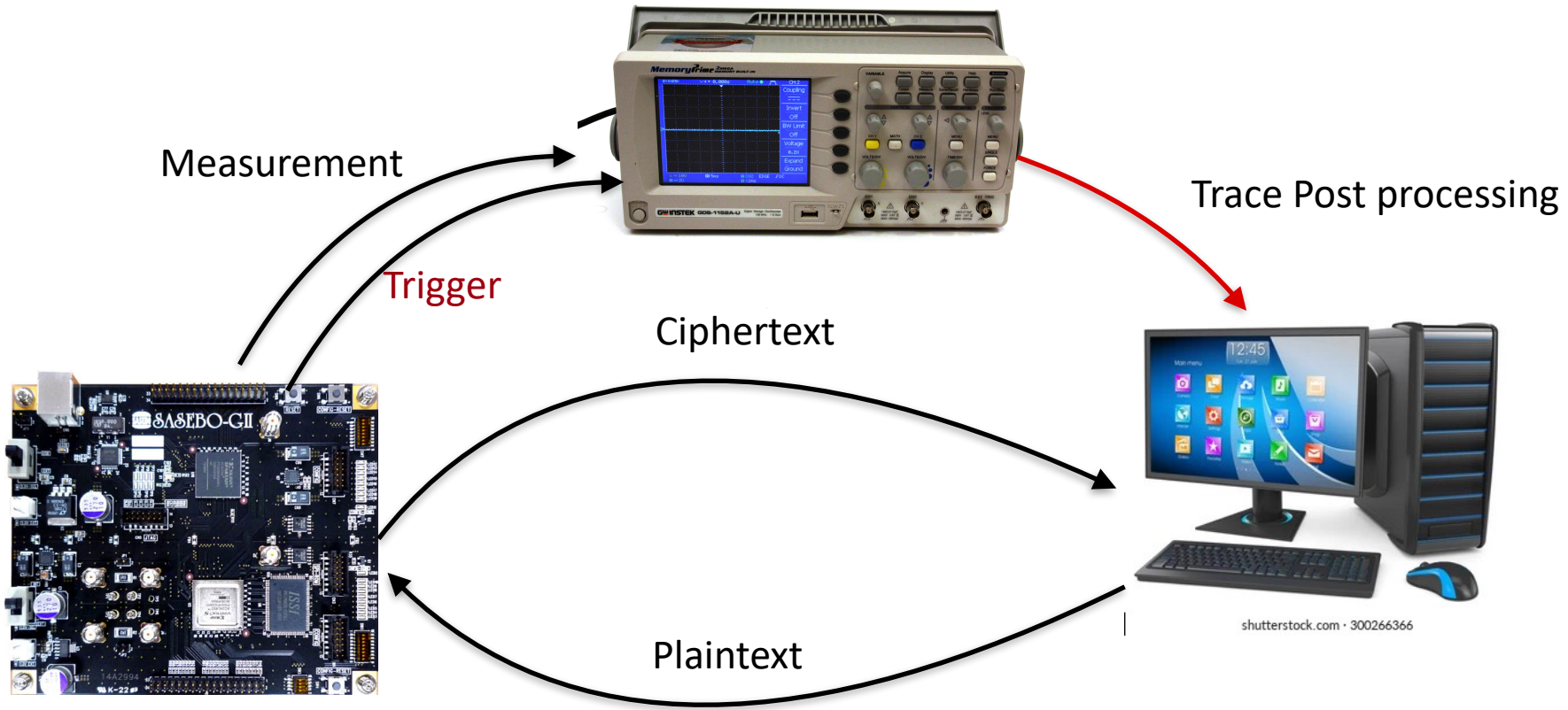
Attacking AES: Attack Complexity

- **What is the attack complexity**
  - Say you are given n number of traces
  - How much further computation you need to perform?

Attacking AES: Attack Complexity

- **Observe**: We are targeting the last round of AES and attacking each S-Box separately.
  - Divide and Conquer
  - Attack complexity: always 2^8
  - Trace complexity: depends
- **Observe**: The attack don’t need plaintext knowledge

Leakage Models


Measurement
Trigger
Trace Post processing
Ciphertext
Plaintext
SASEBO-GII
shutterstock.com · 300266366

Differential Power Analysis

- **Power Trace**: A set of power consumptions across a cryptographic process
  - 1 millisecond operation sampled at 5MHz yield a trace with 5000 points.
- **Measurement:**
  - Usually requires to adhere a signal bandwidth and frequency.
    - We do quite high frequency sampling — e.g. for a 65 Mhz microcontroller or 150 Mhz FPGA implementation, we do sampling at 5 GS/sec.
  - Number of samples is an important point. More samples results in better attack
  - **Trigger**: An important constraint
    - Without reliable trigger, the traces will be **misaligned**.—attack will be difficult
    - Most of the practical implementations does not provide a very reliable trigger
      - So, some realignment of traces are needed
        - Today, DL algorithms are great for this purpose — does automatic realignment.

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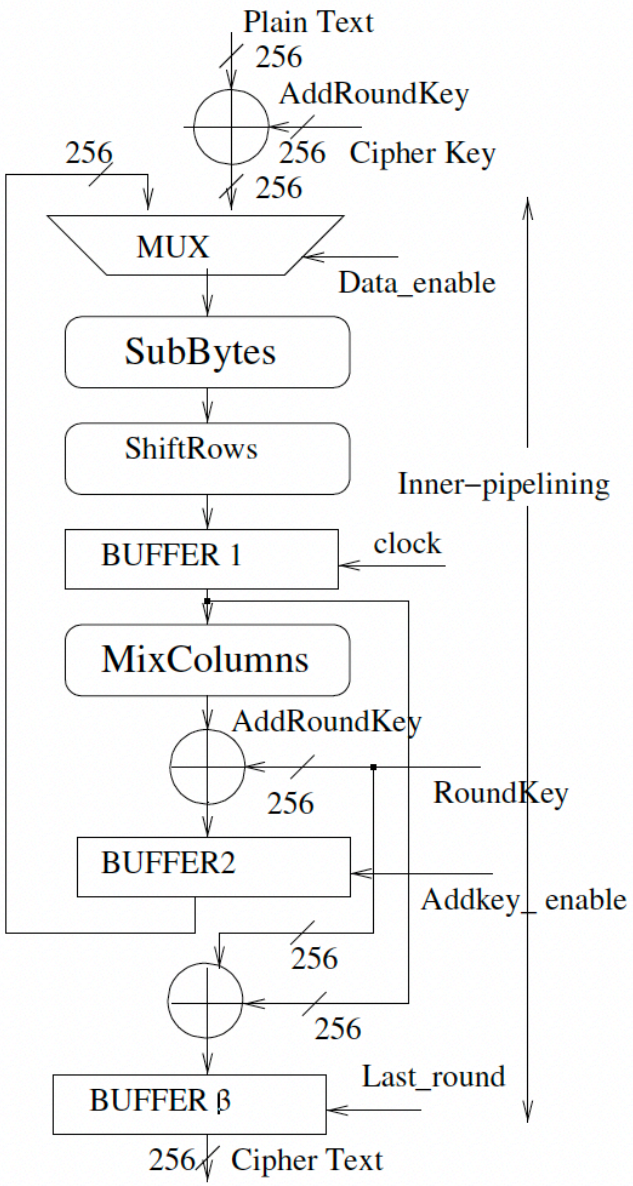

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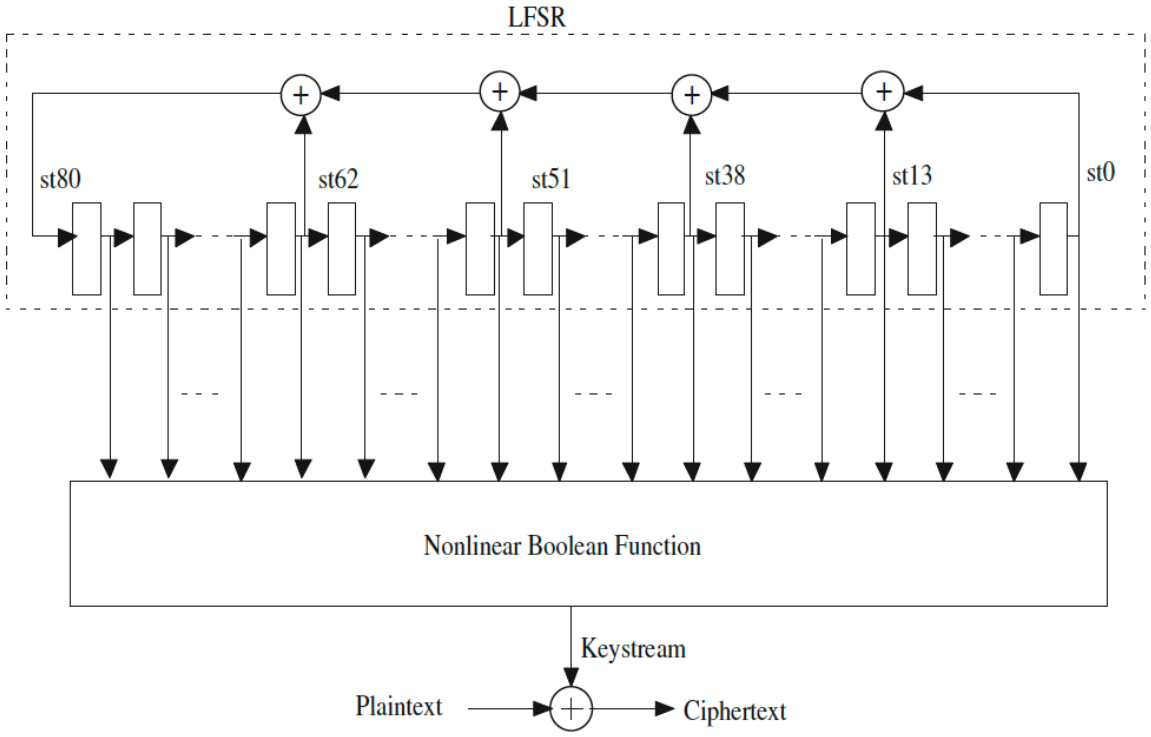
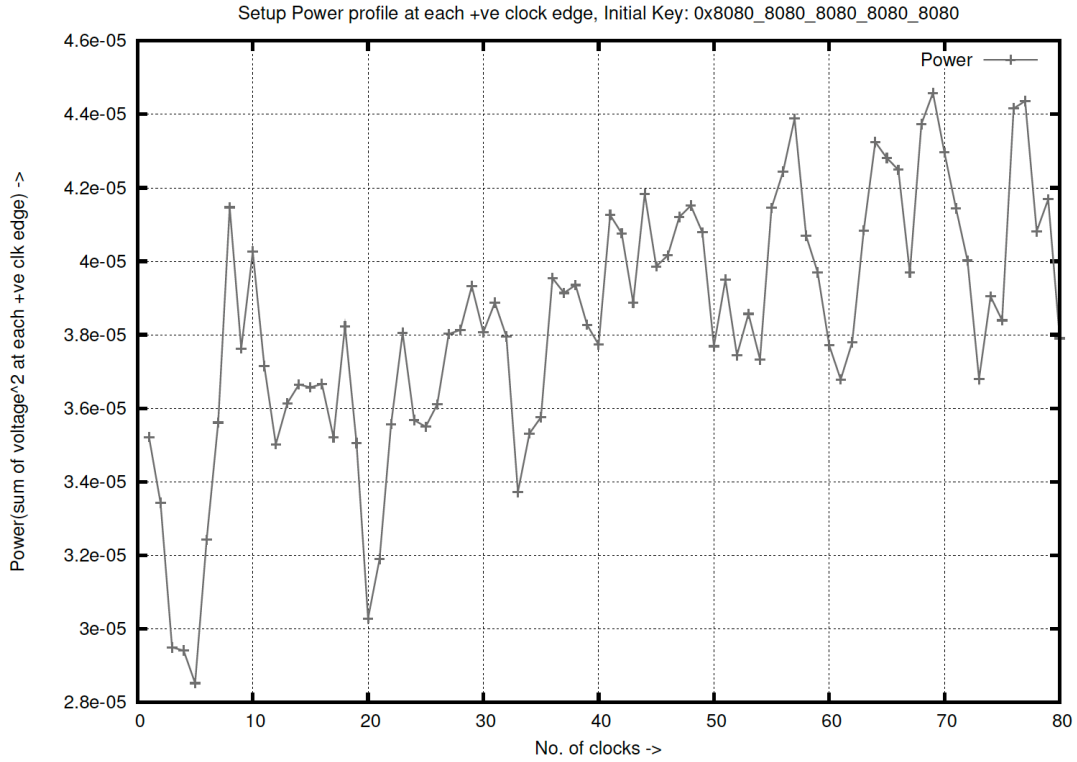
- Let this state be v_t
- Hamming weight is number of 1's in v_t.
- Hamming distance is HW($v_t \oplus v_{t-1}$)..
  - Why?
  - After all power consumption is a result of transition $v_{t-1} \rightarrow v_t$
  - So , HW is relatively inaccurate, but works well for software
    - Registers often start from a specific initialisation value.

Differential Power Analysis

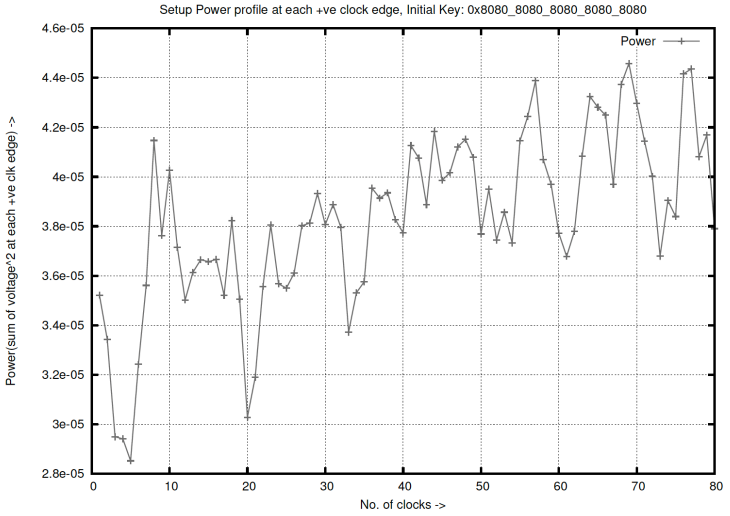


Linear Feedback Shift Register

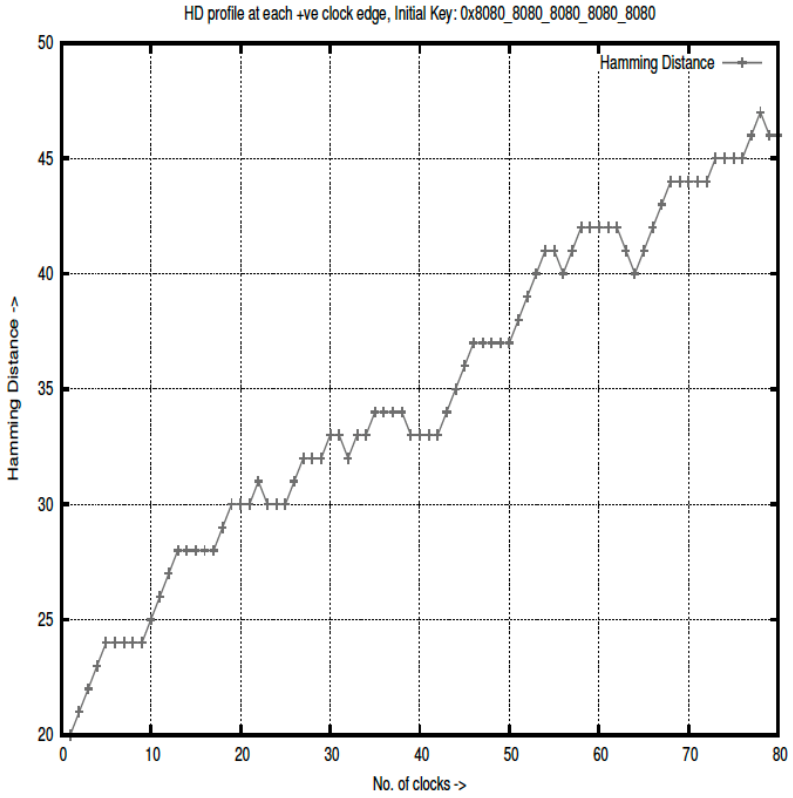


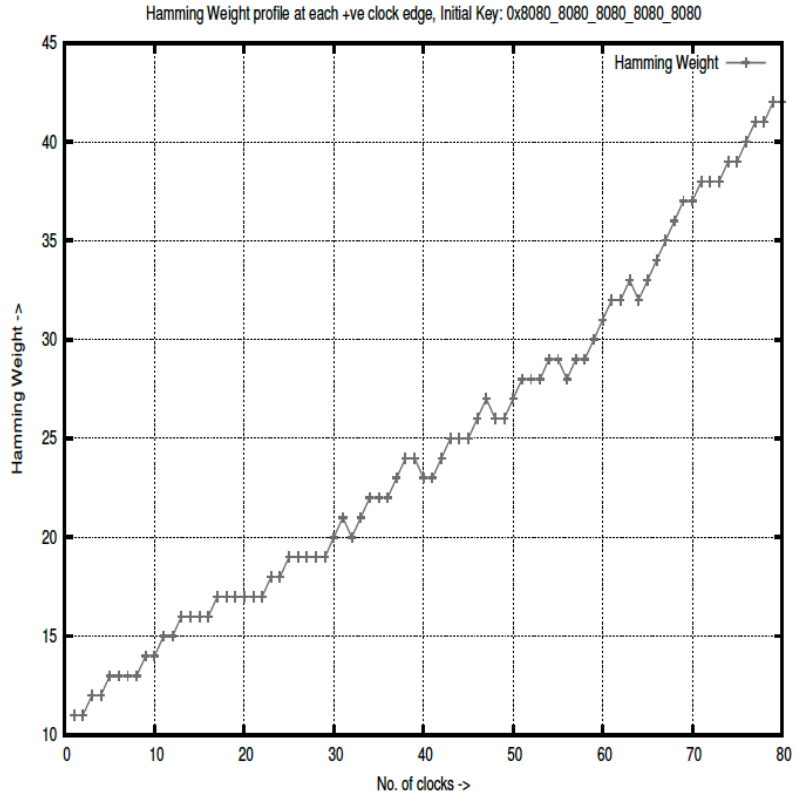
Actual Trace from an FPGA

Differential Power Analysis



Actual Trace from





Actual Trace from an FPGA

Why do Gates Leak?

a	b	y	Energy
0→0	0→0	0→0	$E_{0\to0}$
0→0	0→1	0→0	$E_{0\to0}$
0→0	1→0	0→0	$E_{0\to0}$
0→0	1→1	0→0	$E_{0\to0}$
0→1	0→0	0→0	$E_{0\to0}$
0→1	0→1	0→1	$E_{0\to1}$
0→1	1→0	0→0	$E_{0\to0}$
0→1	1→1	0→1	$E_{0\to1}$
1→0	0→0	0→0	$E_{0\to0}$
1→0	0→1	0→0	$E_{0\to0}$
1→0	1→0	1→0	$E_{1\to0}$
1→0	1→1	1→0	$E_{1\to0}$
1→1	0→0	0→0	$E_{0\to0}$
1→1	0→1	0→1	$E_{0\to1}$
1→1	1→0	1→0	$E_{1\to0}$
1→1	1→1	1→1	$E_{1\to1}$

- Consider an AND gate $y = ab$
- 4 different energy levels due to transition of the gate $E_{0\to0}$, $E_{0\to1}$, $E_{1\to0}$, $E_{1\to1}$
- We estimate $E(q=0)$, and $E(q=1)$ which are the average energy levels when $q=0$, and $q=1$, respectively.
- $E(q=0) = \dfrac{9E_{0\to0} + 3E_{1\to0}}{12}$
- $E(q=1) = \dfrac{3E_{0\to1} + E_{1\to1}}{4}$
- So $E(q=0) \neq E(q=1)$

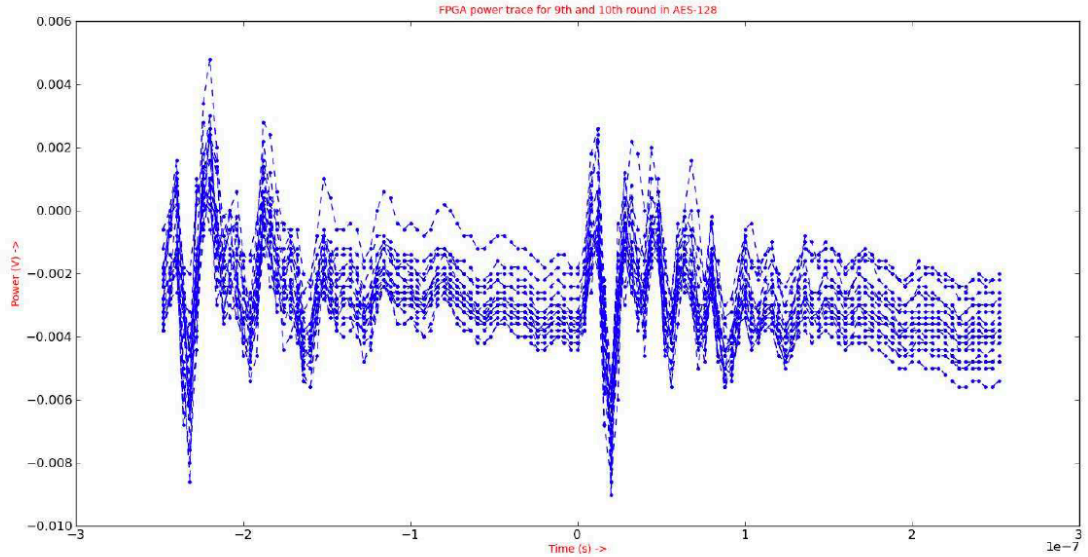
Make Some Noise

Measurement Noise

- **Importance of high-quality measurement**
  - Noise increases the required trace count.

- These fluctuations are due to electrical noise, caused due to power supply, clock generator, conduction and radiation emissions from the components

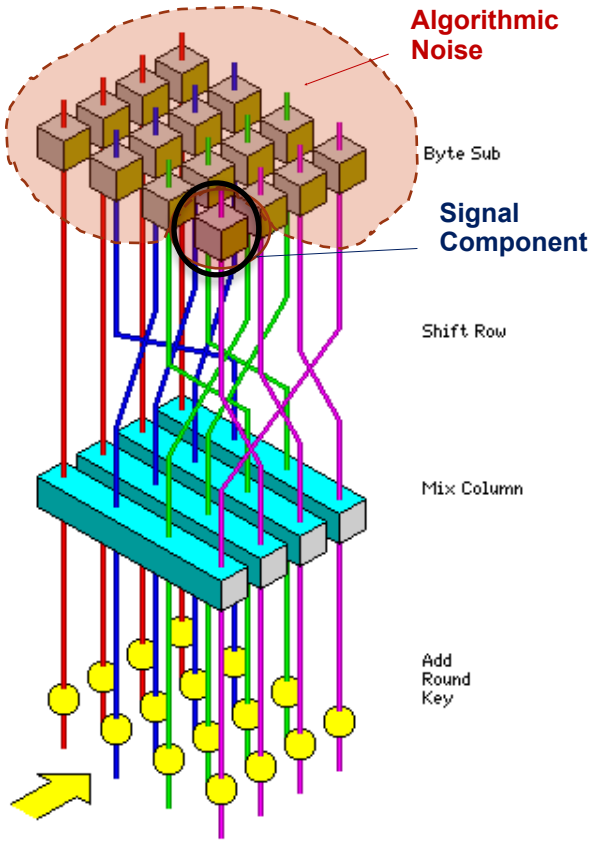
AES-128 Encryption with same plaintext and key, but resulting in different power traces.



Simple Power Model. *Let t denote the time, and $\mathcal{N}(t)$ be a normal distributed random variable which represents the noise components. Let $f(g,t)$ denote the power consumption of gate g at the time t. Then a simplified power model for the power consumption is the function*

$$P(t) = \sum_{g} f(g,t) + \mathcal{N}(t)$$

Algorithmic Noise



Fortunately, our statistical attack can handle both

- Algorithmic or switching noise occurs because of contributions of logic cells to the power consumption, which are not under attack.
- The power trace corresponds to the total power consumption of the circuit.
- However, in the attack we target only a small part (see black circle in fig.) to reveal a portion of the key.
- The power consumption from all the other parts (see blue shaded portion in fig.) form the algorithmic noise.
- This would be more in a parallel implementation, compared to a serialized implementation.

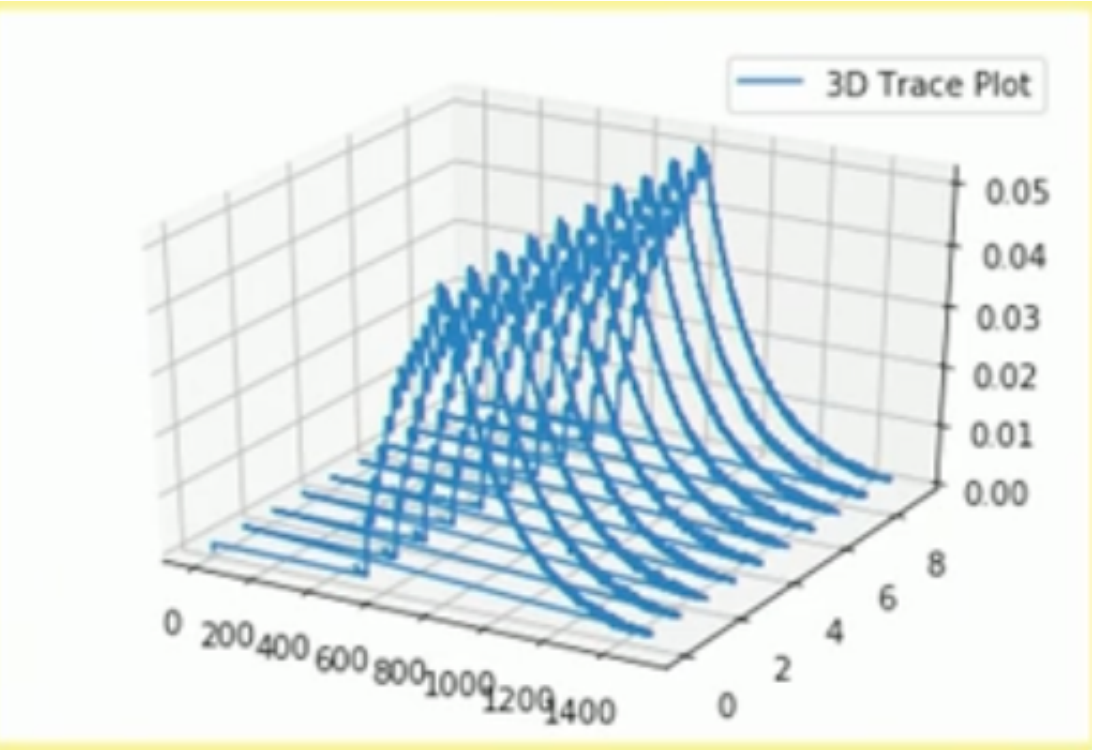
Statistical Analysis of Power Traces

- **Fact**: For fixed operation and fixed value, the power consumption is fixed. But there will be some additive noise.
- **Fortunately, this noise is Gaussian.**
  - Let us assume that the mean is μ and variance is σ^2
  - Noise mainly results in the variance…
- **Recall**: The gaussian probability distribution function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} \qquad x \in \mathbb{R}$$

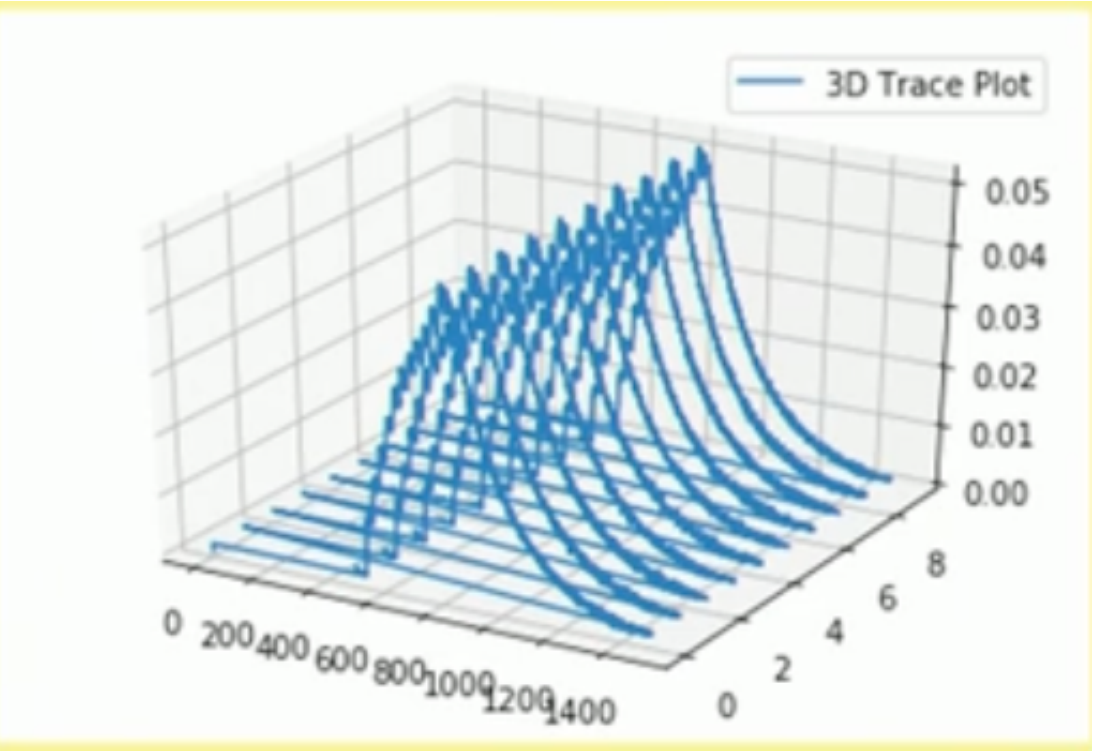
$$X \sim \mathcal{N}(\mu, \sigma)$$

- If $\mu = 0,\ \sigma = 1$, we call it a standard normal distribution.
- The CDF of standard normal is denoted as $\phi(x)$.

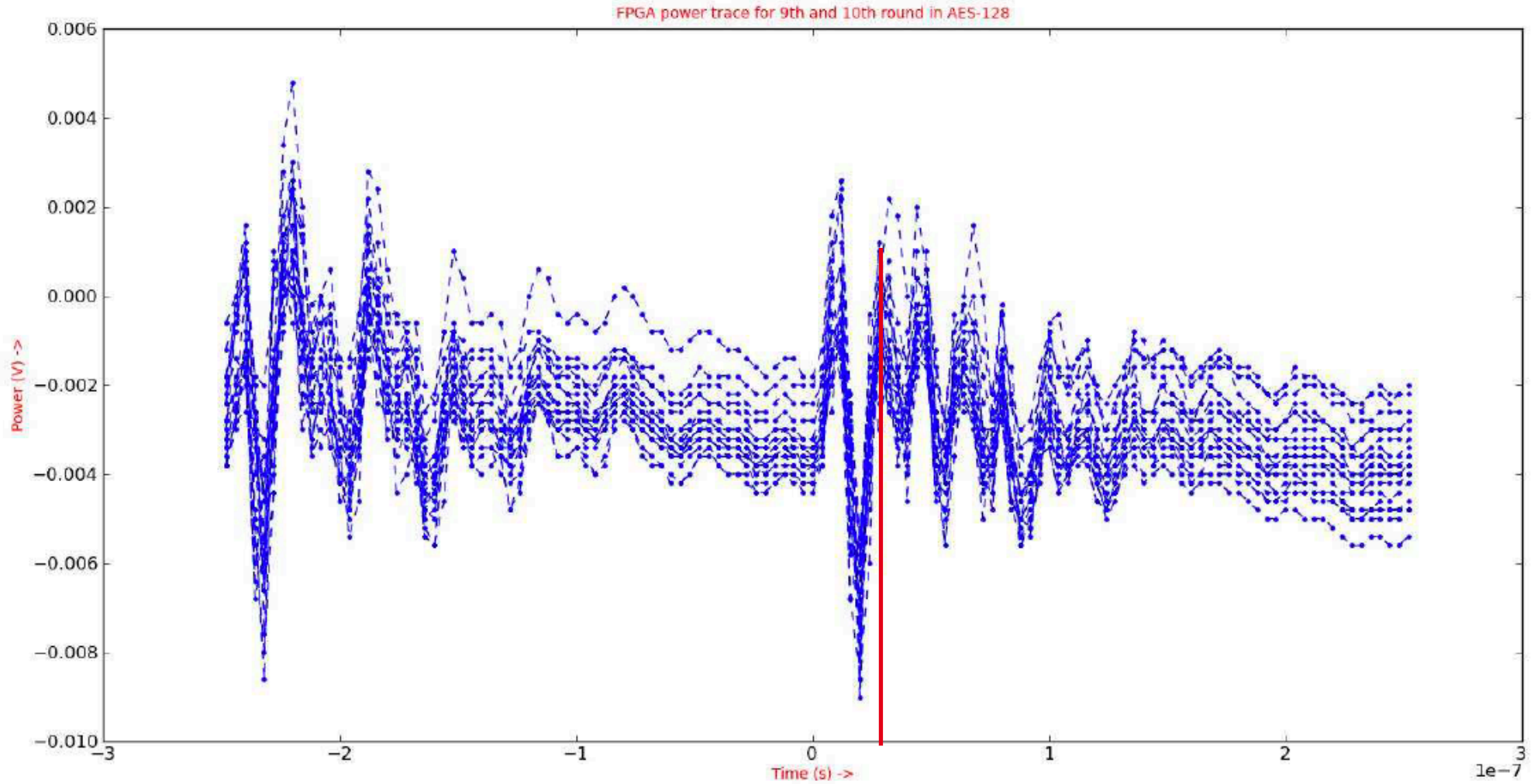


Statistical Analysis of Power Traces

- **Recall**: Sampling distribution
  - An *unbiased estimator* for μ is $\bar{x}$ — the sample mean
  - An unbiased estimator for σ^2 is s^2 — the sample variance (with n-1 correction factor, of course)
  - **Unbiased estimator**: The expected value of the estimator is equal to the true value of the parameter; e.g. $E(\bar{x}) = \mu$

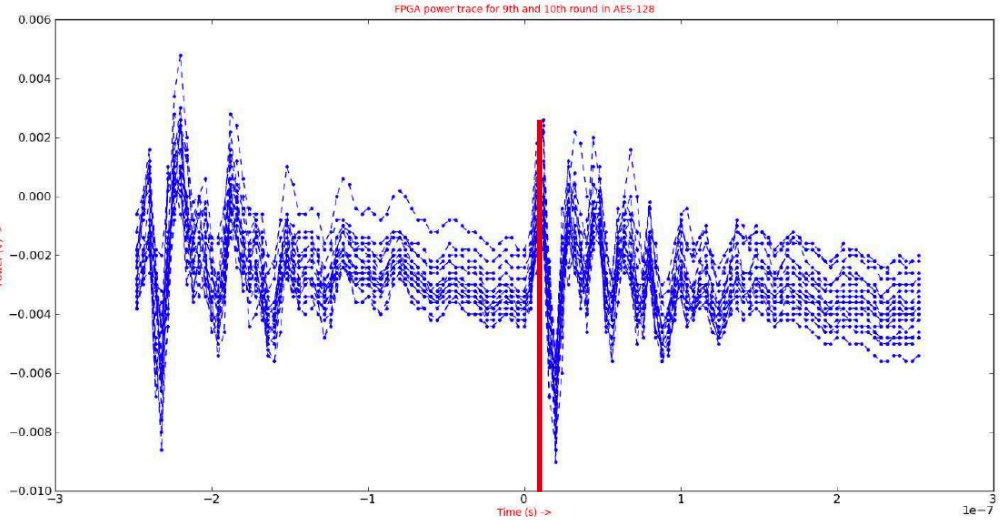


Statistical Analysis of Power Traces

- 

Statistical Analysis of Power Traces

- **A point in the power trace is represented as:**
  - $$P(t) = \sum_{g} f(g,t) + \mathcal{N}(\mu, \sigma, t)$$
- **POI (point of interest) in the trace:** The point(s) in the trace which are the best for distinguishing the correct key from the wrong one.



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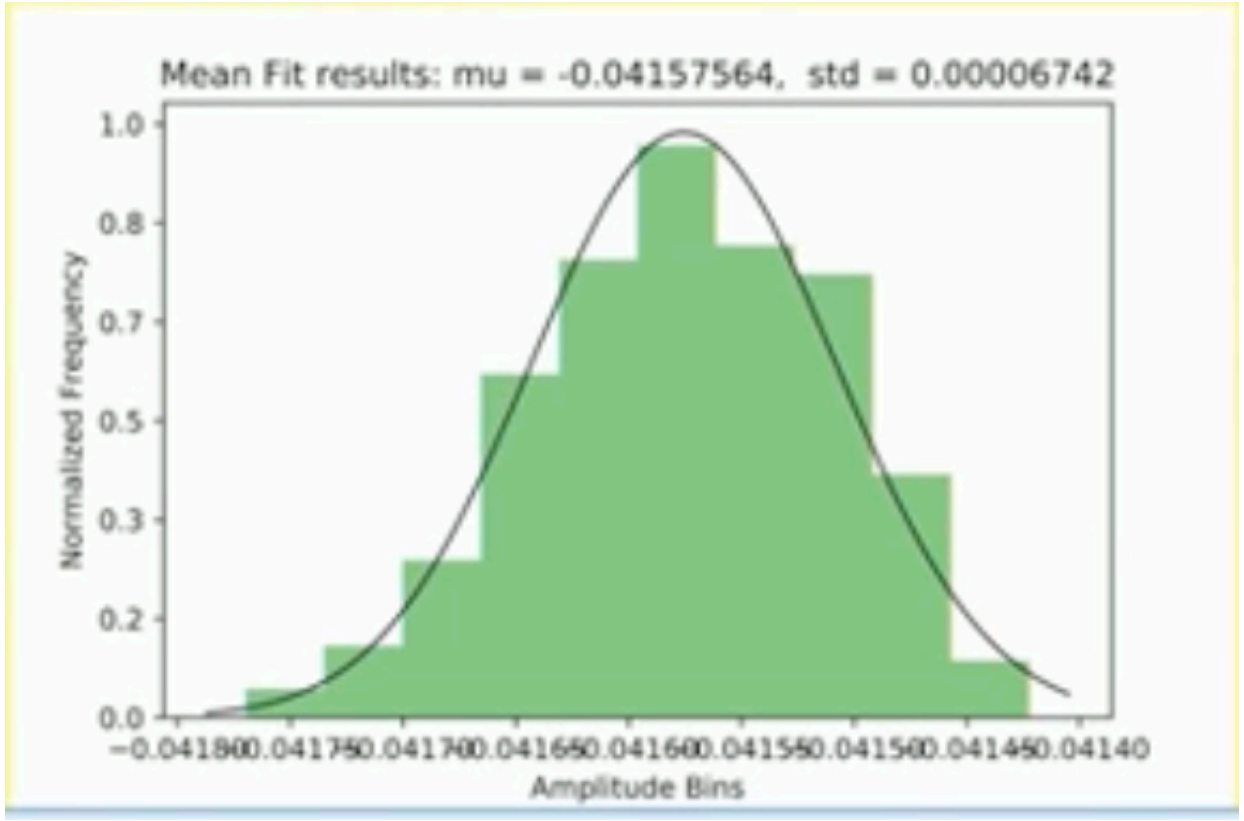


Histogram of a point in power trace for same pt and key

Sampling Distribution and Estimation of Parameters

- **A point in the power trace is represented as:**
  - $$P(t) = \sum_{g} f(g,t) + \mathcal{N}(\mu, \sigma, t)$$
- The question is, how does it affect the attacks??
  - **You cannot determine the exact HW/HD**
- How to manage this noise??
  - **By modelling it**
  - **Taking mean is a good idea as it is done for DoM**

- **<u>Sampling distribution</u>:**
  - **How do we estimate μ, σ?**
  - We can only access some samples from the true distribution and therefore we can only compute the sample mean, variance $\bar{x}, s^2$
  - They are indeed unbiased estimators.
  - But then, there are a few “statistical” points..

Sampling Distribution and Estimation of Parameters

- **Sampling distribution**:
  - **How do we estimate μ, σ?**
  - We can only access some samples from the true distribution and therefore we can only compute the sample mean, variance $\bar{x}, s^2$
  - They are indeed unbiased estimators.
  - But then, there are a few “statistical” points..
  - Each of $\bar{x}, s^2$ has their own distribution… the **sampling distributions**

Sampling Distribution and Estimation of Parameters

- **Sampling distribution**:
  - **Let us consider the sample mean $\overline{X}$ as a random variable**
  - **Note that the population is normally distributed.**
  - It can be shown that $\overline{X} \sim \mathcal{N}(\mu, \sigma/\sqrt{n})$
    - You can prove it !! — try at home
  - **Central Limit Theorem**: Let $X_1, X_2, \cdots X_n$ be independent and identically distributed random variables with $E(X) = \mu < \infty$, and variance $0 < \sigma^2 < \infty$. Then:

$$\lim_{n\to\infty} Pr[Z_n \leq x] = \phi(x), \forall x \in \mathbb{R}$$

$$Z_n = \frac{(X_1 + X_2 + X_3 + \cdots X_n) - n\mu}{\sqrt{n}\sigma} = \frac{\overline{X} - n\mu}{\sqrt{n}\sigma}$$

This means, whatever be the population's distribution, the sample mean will approximately follow the normal distributions for sufficiently large number of sample (n > 30, typically).

- **Law of large numbers**: $\lim_{n\to\infty} \frac{X_1 + X_2 + \cdots X_n}{n} \to \mu$, that is $\overline{X}$ is unbiased estimator.

Sampling Distribution and Estimation of Parameters

- **<u>Sampling distribution</u>:**
  - So, the more traces you take, the average becomes closer to the actual mean.
  - That means, if you have noise with $\mu = const$, with more traces you can significantly reduce its impact.
  - **But, how do you know that how many traces take you closest to the mean???**

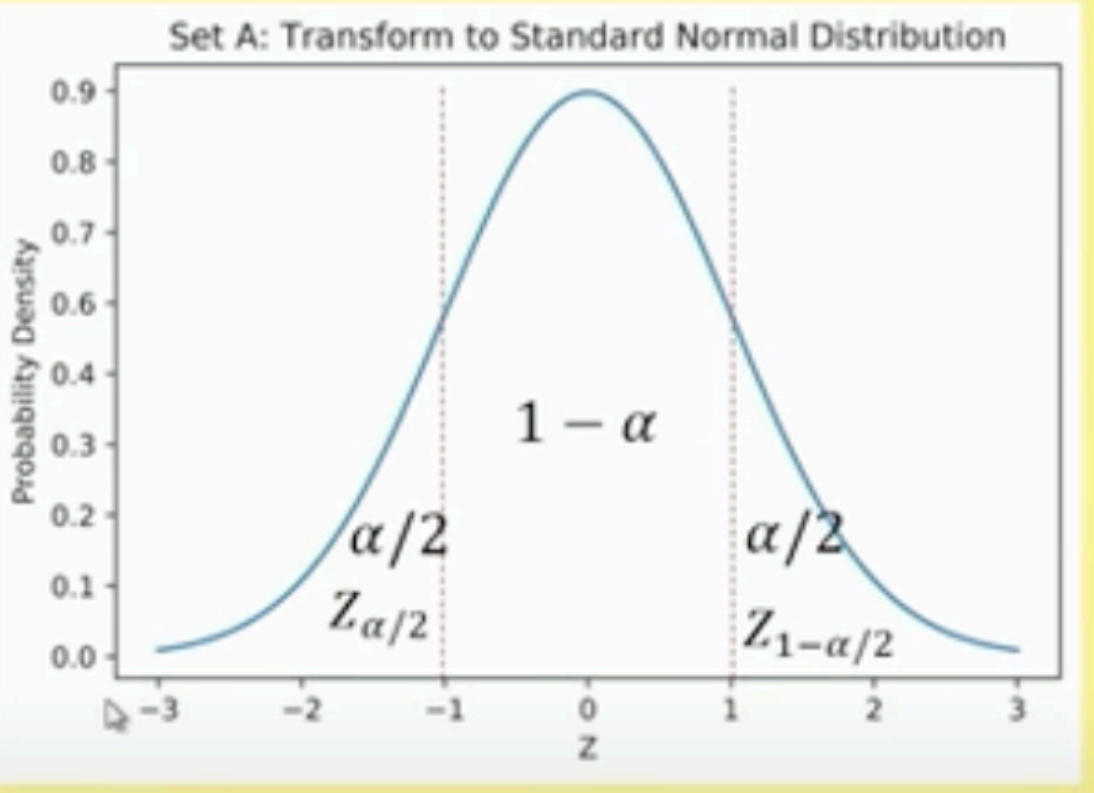
Hypothesis Testing

- **Confidence Interval**
  - **Determines, how close a certain parameter estimate is to its actual value**
  - When we say that we have a .99 confidence interval for μ, actual μ lies in that interval, and that happens 99% of the time you calculate $\bar{x}$ and the interval.
  - **How do we find out these intervals?**
    - We shall see soon..

- **Hypothesis Testing**
  - We define a hypothesis and test whether it is true or not depending on the samples
  - Suppose we hypothesize that $\mu = \mu_0$
    - You set a **null hypothesis:** $H_0 : \mu = \mu_0$
    - You set an **alternative hypothesis:** $H_1 : \mu \neq \mu_0$
    - **Two hypotheses must be exclusive.**
    - **It can be something else as well, such as** $H_0 : \mu_1 \neq \mu_2$

- **Normal to Standard Normal:** If $X \sim \mathcal{N}(\mu, \sigma)$ then $Z = \dfrac{X - \mu}{\sigma} \sim \mathcal{N}(0,1)$

Standard Normal Distribution



- Let us define an as an interval $[-Z_{\alpha/2}, Z_{\alpha/2}]$
- $P(Z \leq Z_\alpha) = \phi(Z_\alpha) = \alpha$
- **Also, we can see that:** $Z_\alpha = -Z_{1-\alpha}$
- The probability that a randomly sampled value of Z will lie in this interval is $1-\alpha$ — $P(-Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}) = 1-\alpha,$

- The normal distribution tables provide numerically calculated values for $\phi(Z_\alpha)$ for different α

Confidence Interval for μ

$$\overline{X} \sim \mathcal{N}(\mu, \sigma/\sqrt{n})$$
$$\implies Z = (\overline{X} - \mu)\frac{\sqrt{n}}{\sigma}$$

- Therefore,

- $P(Z_{\alpha/2} \leq (\bar{X} - \mu) \cdot \frac{\sqrt{n}}{\sigma} \leq Z_{1-\alpha/2})$
  $= P(\frac{\sigma}{\sqrt{n}} Z_{\alpha/2} \leq (\bar{X} - \mu) \leq \frac{\sigma}{\sqrt{n}} Z_{1-\alpha/2})$
  $= P(\bar{X} - \frac{\sigma}{\sqrt{n}} Z_{1-\alpha/2} \leq \mu \leq \bar{X} - \frac{\sigma}{\sqrt{n}} Z_{\alpha/2}).$
- Using, $Z_{\alpha/2} = -Z_{1-\alpha/2}$, thus we continue:
  $= P(\bar{X} - \frac{\sigma}{\sqrt{n}} Z_{1-\alpha/2} \leq \mu \leq \bar{X} + \frac{\sigma}{\sqrt{n}} Z_{1-\alpha/2}).$

- Given $\overline{X} \sim \mathcal{N}(\mu, \sigma/\sqrt{n})$
- We need to compute "how good is the estimate of μ for a given number of samples (traces) *n*"
- If we can estimate μ well, then we actually average out the noise and get close to the actual power model (HW/HD/ or something else).
- **Overall, we need to compute a confidence interval of μ, based on the sample mean $\overline{X}$.**
- Informally, that means, we do not know the exact value of μ, but we can say (based on $\overline{X}$), what range it lies.
- $[\overline{X} - \frac{\sigma}{\sqrt{n}} Z_{1-\frac{\alpha}{2}}, \overline{X} + \frac{\sigma}{\sqrt{n}} Z_{1-\frac{\alpha}{2}}]$ is the $(1-\alpha)$ **confidence interval** for μ.
- α is called the **error probability**

Confidence Interval for μ

- Overall, μ lies in the interval

  $[\overline{X} - \frac{\sigma}{\sqrt{n}} Z_{1-\frac{\alpha}{2}}, \overline{X} + \frac{\sigma}{\sqrt{n}} Z_{1-\frac{\alpha}{2}}]$ with

  probability $1 - \alpha$
- Say $1 - \alpha = 0.99$, **then it means that if you take say 100 sets of n-samples, μ will lie in the aforementioned range 99 times.**
- It is the “confidence” on the interval finding mechanism, not in the found interval.

Hypothesis Test

- Suppose, we want to test if $\mu = \mu_0$
  - **null hypothesis:** $H_0 : \mu = \mu_0$
  - **alternative hypothesis:** $H_1 : \mu \neq \mu_0$
- μ is estimated from the sample mean $\overline{X}$
- We check
  - If $|\overline{X} - \mu_0|$ is greater than some predefined constant *c*, we reject $H_0 : \mu = \mu_0$**.**
  - Else, we accept $H_0 : \mu = \mu_0$
- **False positive (Type I error):**
  - The null hypothesis is correct but got rejected;
  - $P(|\overline{X} - \mu_0| > c) = \alpha$ — is therefore called the error probability and $1 - \alpha$ is the level of confidence.
- **False negative (Type II error)**: The null hypothesis is wrong but got accepted.

Estimating the Number of Traces to Estimate μ

- Suppose, we want to estimate the mean with precision $c = 0.01$

$$P(|\bar{X} - \mu_0| > c) = \alpha,$$

$$P(|\bar{X} - \mu_0| > c) = P\left(|\bar{X} - \mu_0|\frac{\sqrt{n}}{\sigma} > c\frac{\sqrt{n}}{\sigma}\right) = P\left(|Z| > c\frac{\sqrt{n}}{\sigma}\right) = 2P\left(Z > c\frac{\sqrt{n}}{\sigma}\right) = \alpha$$

=> $$P\left(Z > c\frac{\sqrt{n}}{\sigma}\right) = \frac{\alpha}{2} \Rightarrow \mathbf{c\frac{\sqrt{n}}{\sigma} = Z_{1-\alpha/2}}$$

- Finally, $$\boldsymbol{n = \frac{\sigma^2}{c^2} Z^2_{1-\alpha/2}}$$

Estimating the Number of Traces for DoM

- Suppose, $X \sim \mathcal{N}(\mu_X, \sigma)$, $Y \sim \mathcal{N}(\mu_Y, \sigma)$ — let us assume the variance is the same. Let us also assume that we have n traces.
  - $\overline{X} - \overline{Y} \sim \mathcal{N}(\mu_X - \mu_Y, \sigma\sqrt{\frac{n+n}{n^2}})$
  - $Z = \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sigma\sqrt{\frac{2}{n}}}$ is the corresponding standard normal variable.

Estimating the Number of Traces for DoM

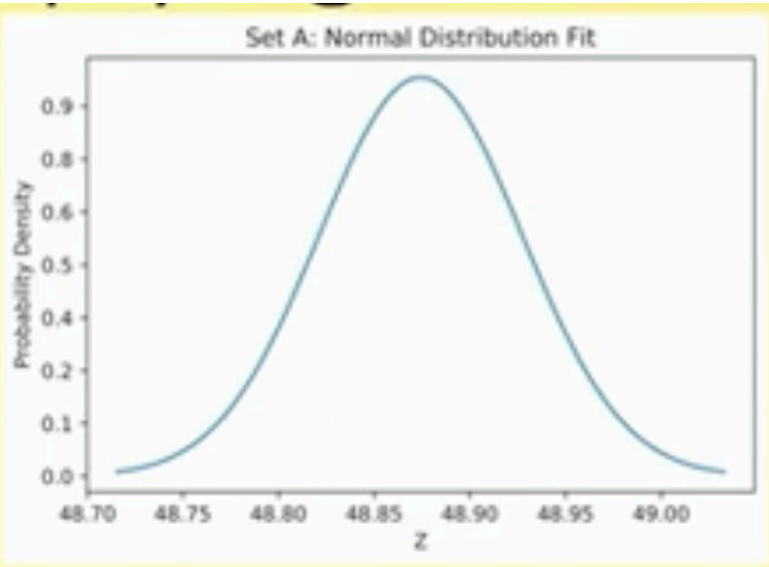
- Confidence interval for $(\mu_X - \mu_Y)$ for known σ:

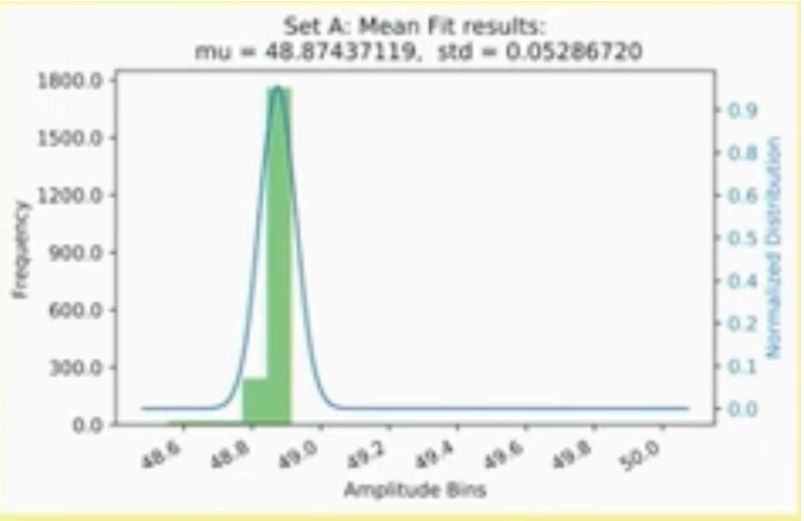
$$P(Z_{\alpha/2} \leq Z \leq Z_{1-\alpha/2}):$$
$$[\bar{X} - \bar{Y} - \frac{\sigma}{\sqrt{n/2}} Z_{1-\alpha/2}, \bar{X} - \bar{Y} + \frac{\sigma}{\sqrt{n/2}} Z_{1-\alpha/2}]$$

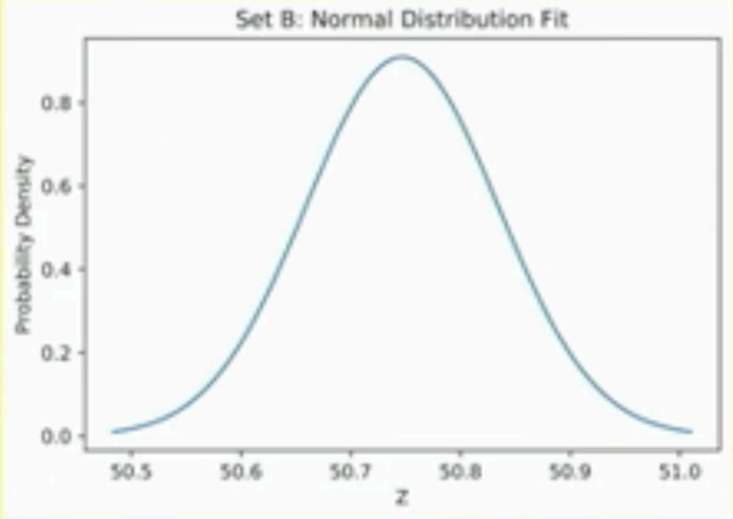
  - **Null Hypothesis:** $(\mu_X - \mu_Y) = 0$, **Alternate Hypothesis:** $(\mu_X - \mu_Y) \neq 0$
  - **We test if** $Z = \dfrac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sigma\sqrt{\frac{2}{n}}}$ is in the critical region.
  - **Therefore,**

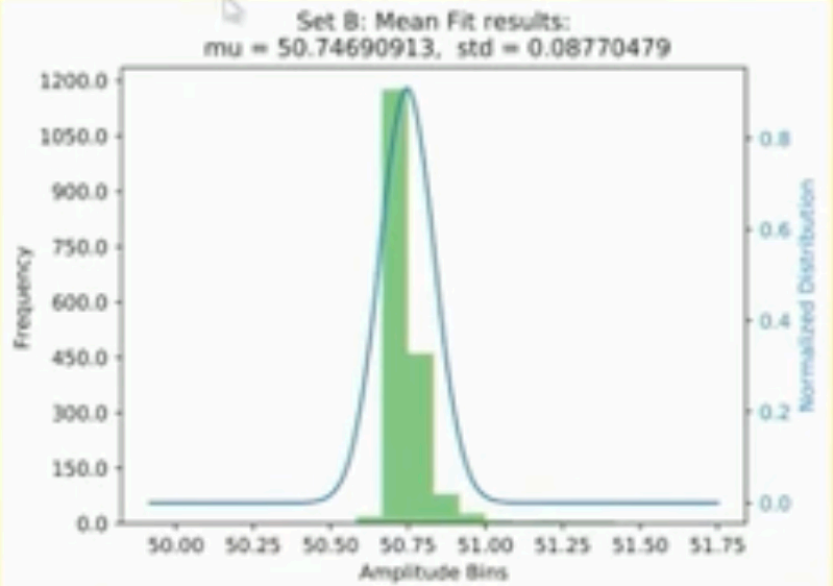
$$2P\left[\frac{\bar{X}-\bar{Y}}{\sigma\sqrt{\frac{2}{n}}} > \frac{c}{\sigma\sqrt{\frac{2}{n}}}\right] = \alpha \Rightarrow \frac{c}{\sigma\sqrt{\frac{2}{n}}} = Z_{1-\alpha/2} \Rightarrow \boldsymbol{n = \frac{2\sigma^2}{c^2} Z^2_{1-\alpha/2}}$$

Estimating the Number of Traces for DoM

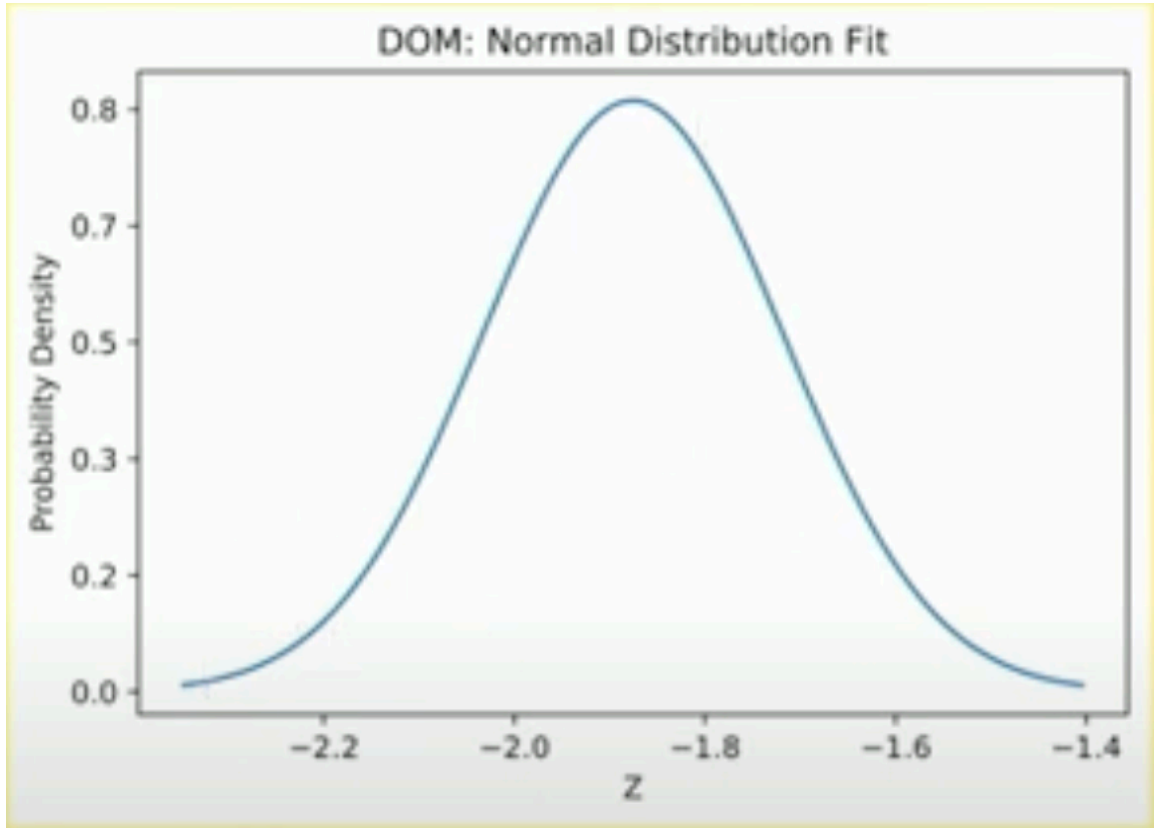


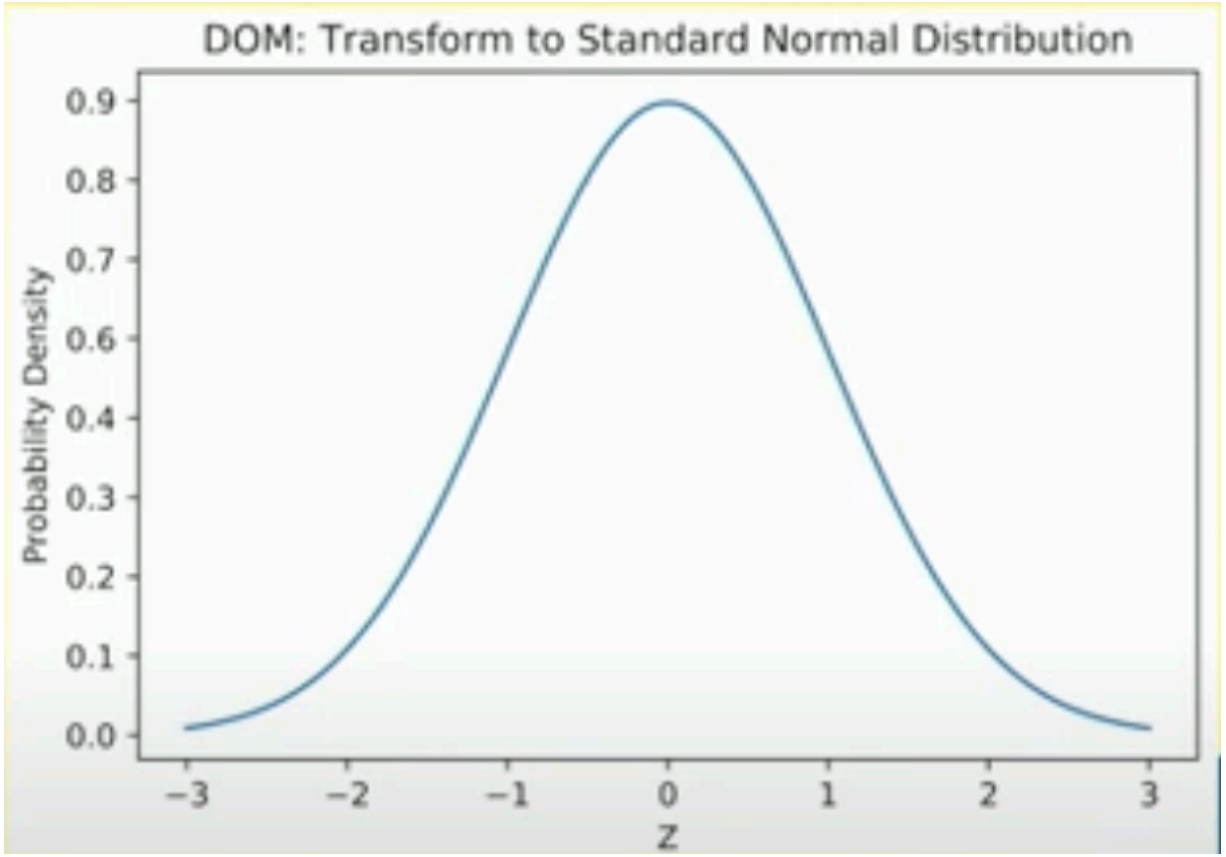






Estimating the Number of Traces for DoM





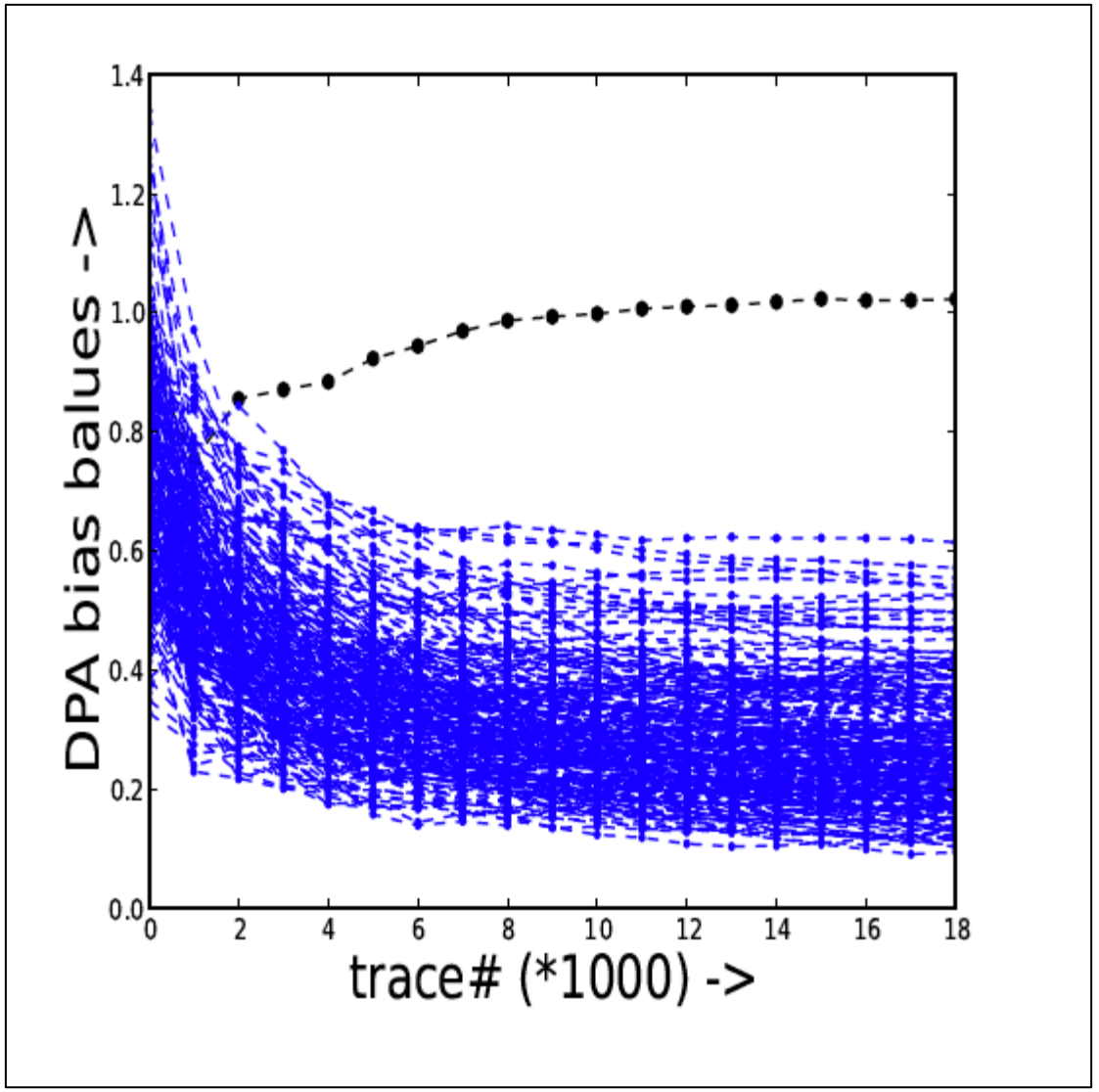
Estimating the Number of Traces for DoM

With around 4296 traces, with equal proportions going to Set X and Y, we have the following observations:

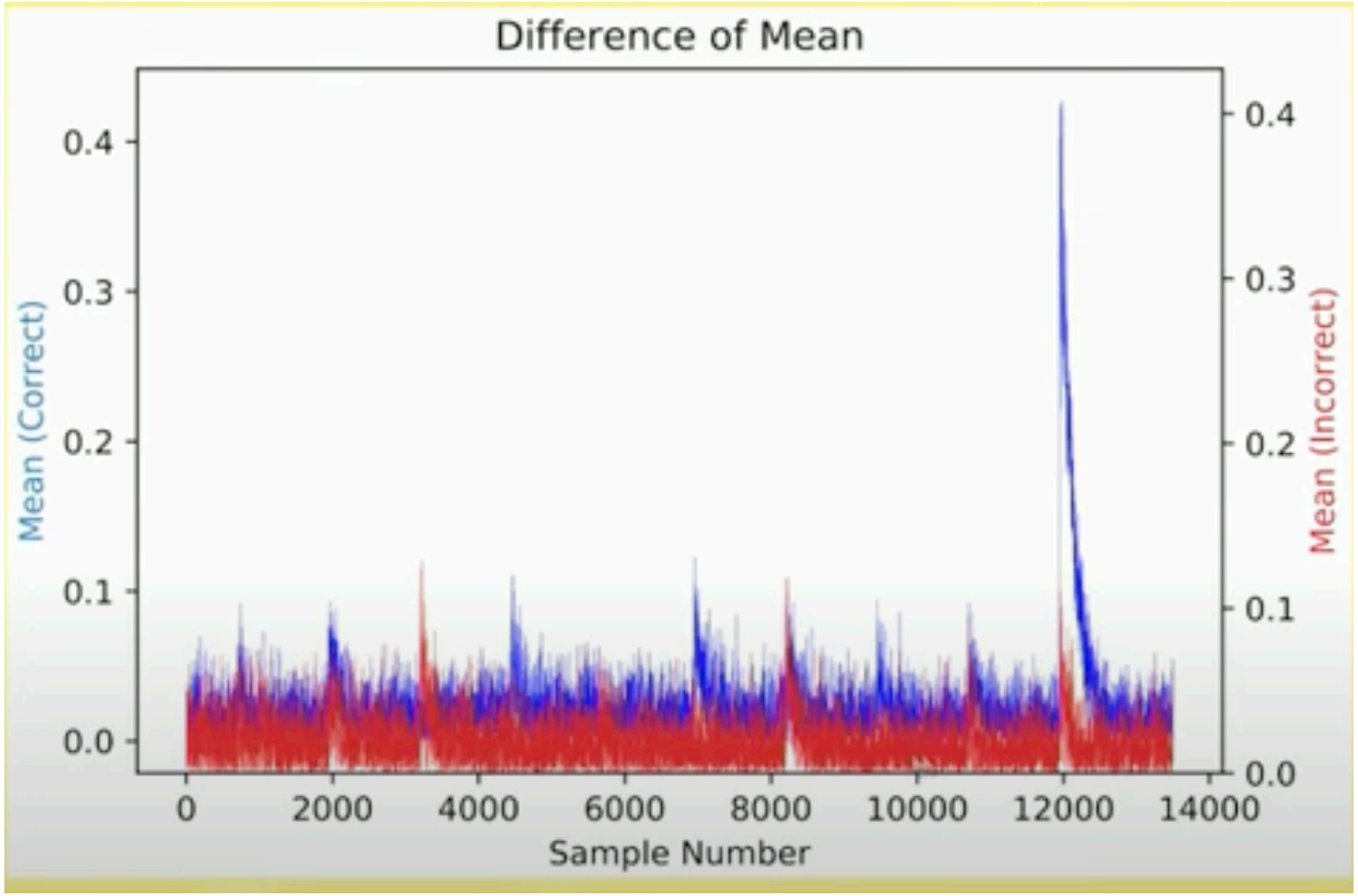
$\sigma = 0.07$. We set $c = 0.01$. For a confidence of $(1 - \alpha) = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow Z_{1-\alpha/2} = Z_{0.975} = 1.96$.

Thus, estimated number of traces=$\frac{2(0.07)^2}{(0.01)^2} 1.96^2 = 377$.

Estimating the Number of Traces for DoM



Estimating the Number of Traces for DoM

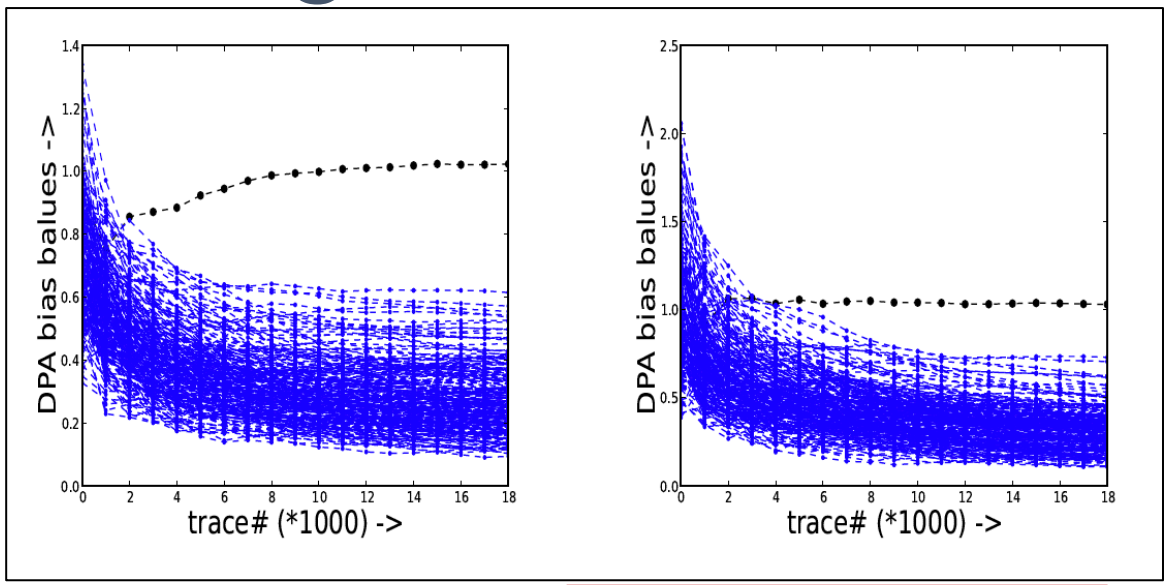


Simulating the Power Traces

- Let NSample denote the number of randomly chosen plaintexts to be encrypted using wlog. the rolled AES cipher.
- In such an architecture, a register is updated by the output of the AES round every encryption.
- The power trace is stored in the array sample[NSample][NPoint], where NPoint is corresponding to the power consumption after each round of encryption.
- For each power trace, we also store the corresponding ciphertexts in the array Ciphertext[NSample]
- We simulate the power traces by using the power model (Hamming weight or Hamming Distance) on the state of the register before the encryption, then followed by that after initial key addition, then after each of the 9 rounds, and finally the cipher state.
- Thus NPoint is 12 and we choose NSample as 8000.

Algorithm for Performing DPA

```
Input: sample[NSample][NPoint],Ciphertext[NSample]
Output: biasKey[NKey],biasIndex[NKey]
1  for (cipher = 0; cipher < NCipher; cipher++) do
2      partialCipher = inverseSBox[cipher ⊕ key]
3      if (partialCipher & 1) then
4          for (j = 0; j < NPoint; j++) do
5              sumBin1[j] += sample[cipher][j]
6              countBin1 += freqSample[cipher]
7          end
8      end
9      else
10         for (j = 0; j < NPoint; j++) do
11             sumBin0[j] += sample[cipher][j]
12             countBin0 += freqSample[cipher]
13         end
14     end
15 end
16 bias = 0
17 for (j = 0; j < NPoint; j++) do
18     meanDiff[j] = sumBin0[j]/countBin0 - sumBin1[j]/countBin1
19     if (bias < abs(meanDiff[j])) then
20         bias = abs(meanDiff[j])
21         index = j
22     end
23     biasKey[key] = bias
24     biasIndex[key] = index
25 end
```



DPA on first key byte of 10th Round of AES
Power is Simulated by Hamming Weight of the Registers after each Round

Power is Simulated by Hamming Weight of the Registers after each Round,with superimposed Gaussian Noise.

DOM and Power Model

- The above attack was demonstrated using Hamming Weight Power Model.
- However, it can be easily adapted for a Hamming Distance power model.
  - Then the classification of the power traces would be based on the change of transitions of the target bit across two successive clock cycles.
  - Rest of Attack is same.

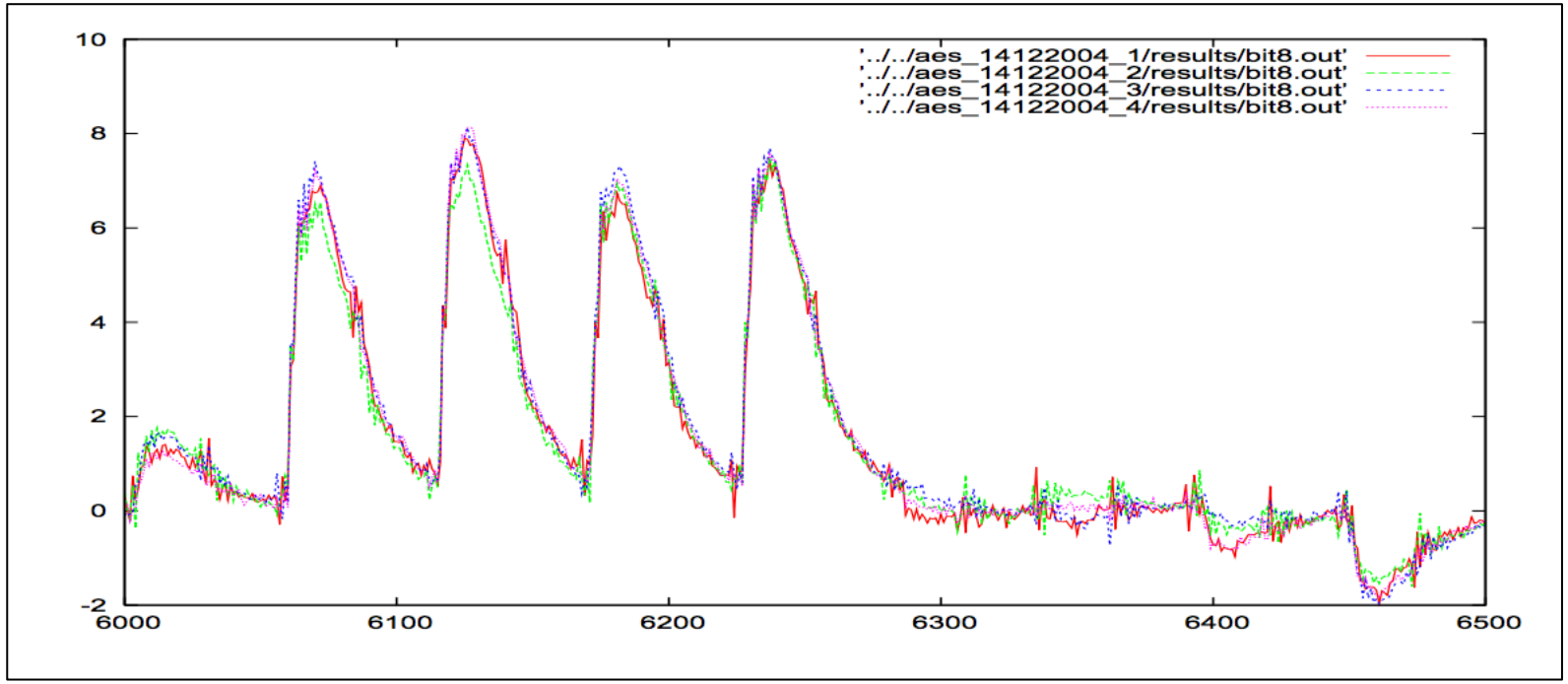
Stochastic Power Models

- The DoM technique for DPA is non-profiled.
- On the contrary, there could be another kind of side channel attacks which fall into the category of profiled attacks.
- In profiled attacks, one needs to have access to a pair of identical devices:
  - Target: limited control and running cipher with fixed key.
  - Profiler: full knowledge and control of the input and keys.
- Profiling can help to stochastically learn the power model more accurately.

An Example of Such a Model

- Consider, a target byte say in the last AES round.
  - $\mathscr{P}\left(\phi\left(x,k\right)\right) = b_0 + \Sigma_{i=1}^{8} b_i(g_i(\phi\left(x,k\right))$
- Here $\phi(x,k) = S^{-1}(x \oplus k)$,
  - x, k are the portions of the cipher and the portion of the key guessed in the last round
  - S^{-1} stands for Inverse Sub Bytes (or inverse of the last round S-Box)
  - g_i extracts the i^{th} bit of the state $\phi(x,k)$.
- In the profiling phase, we keep the deterministic part defined by $\phi(x,k)$ constant.
- However, because of noise we have to solve an over-defined system of equations to compute the values of the $b_i's$.

Coefficient b_8 varying with time t.



4 different keys used.
The deviation of the co-efficient with time shows that Hamming Distance is not the best power model.
However, we need a profiling step.

In this case, 2000 equations were solved (ie experiments were done with 2000 values of x)

Ref: Werner Schindler, Kerstin Lemke, Christof Paar, A Stochastic Model for Differential Side Channel Cryptanalysis, CHES 2005.

Correlation Power Attack (CPA)

- Like DoM based DPA, CPA also relies on targeting an intermediate computation, typically the input or output of an S-Box.
- These input values are computed from a known value, say the ciphertext and a portion of the key, which is guessed.
- The power model is then subsequently applied to develop a hypothetical power trace of the device for a given input to the cipher.
- These hypothetical power values are then stored in a matrix for several inputs and can be indexed by the known value of the ciphertext and the guessed key byte.
- This matrix is denoted as H, the hypothetical power matrix.

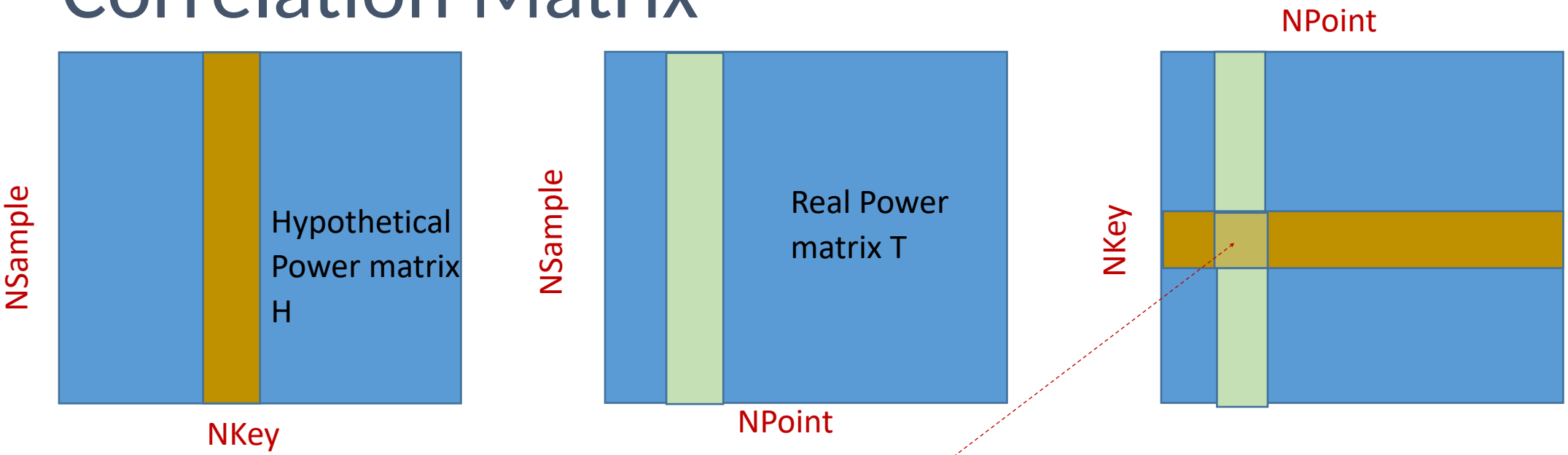
Correlation Power Attack (CPA)-Contd.

- The attacker also observes the actual power traces, and stores them in a matrix for several inputs.
- The actual power values can be indexed by the known value of the ciphertext and the time instance when the power value was observed.
- This matrix is denoted by T, the real power matrix.
- It may be observed that one of the columns of the matrix H corresponds to the correct key k*.
- CPA tries to compute the **similarity** between the columns of the matrix H and the columns of the matrix T, to distinguish k* from rest: similarity computed by Pearson's Correlation, usually.

Computing Correlation Coefficient for Simulated Power Traces for AES

- Like before, we simulate the power profile for the iterative AES, this time by using Hamming Distance power model applied on the state registers updated after each round.
- The real power matrix is stored in the array trace[NSample][NPoint]
  - NSample: Number of Power Traces acquired
  - NPoint: Time instances for which the power values are observed. Here NPoint is 12.

Correlation Matrix

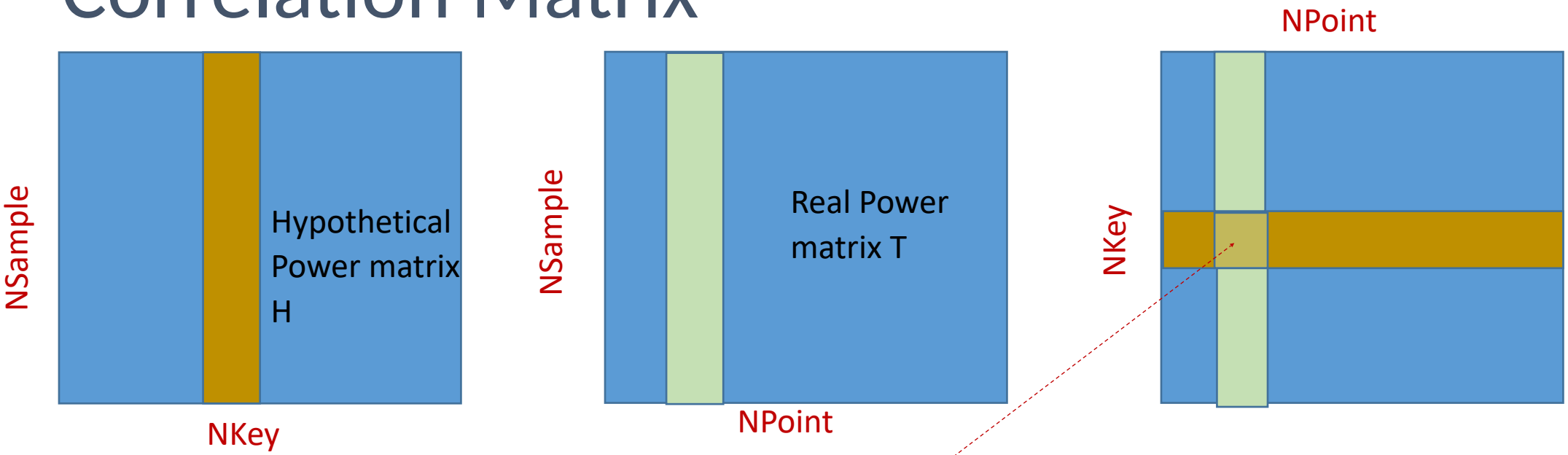


$$C[i][j] = \frac{\Sigma_{k=0}^{NSample}(hPower[i][k] - meanH[i])(trace[j][k] - meanTrace[j])}{\Sigma_{k=0}^{NSample}\left(hPower[i][k] - meanH[i]\right)^2 \Sigma_{k=0}^{NSample}\left(trace[j][k] - meanTrace[j]\right)^2}$$

Computing the Correlation Matrix

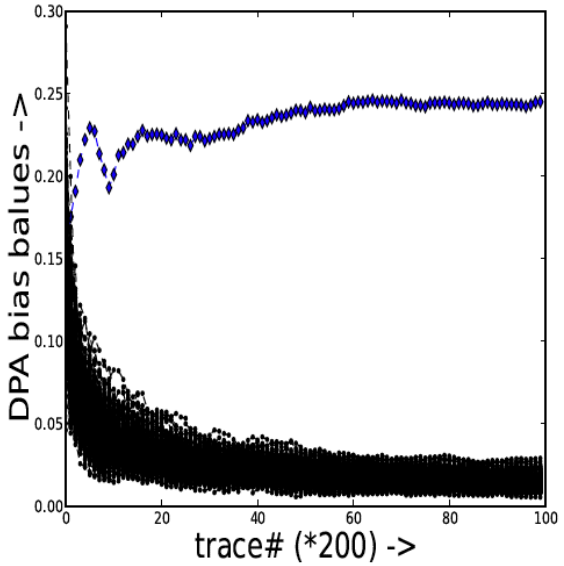
- Actual Power values for all the NSample encryptions are stored in the array trace[NSample][NPoint].
  - However, reflect as we are calculating hypothetical power using HD, the trick which we applied before for storing the traces compactly will not work.
  - Attacker first scans each column of this array and computes the average, and stores in meanTrace[NPoint].
- Likewise, the hypothetical power is stored in the array hPower[NSample][NKey].
  - Attacker scans each column and stores in the array meanH[NKey]

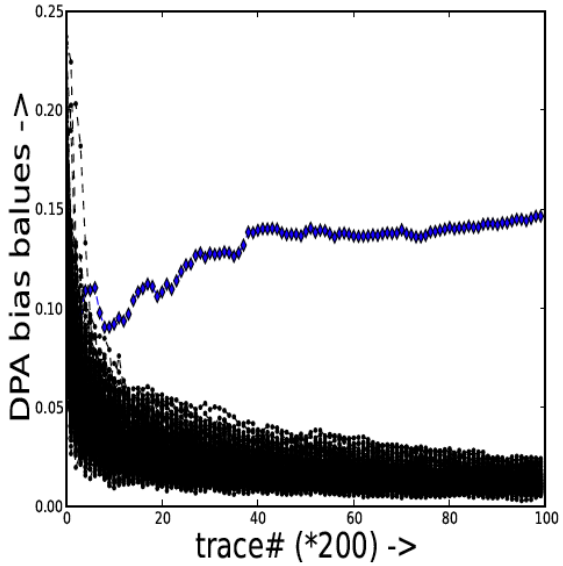
Correlation Matrix



$$C[i][j] = \frac{\Sigma_{k=0}^{NSample}(hPower[i][k] - meanH[i])(trace[j][k] - meanTrace[j])}{\Sigma_{k=0}^{NSample}\left(hPower[i][k] - meanH[i]\right)^2 \Sigma_{k=0}^{NSample}\left(trace[j][k] - meanTrace[j]\right)^2}$$

Experimental Results on Simulated Traces

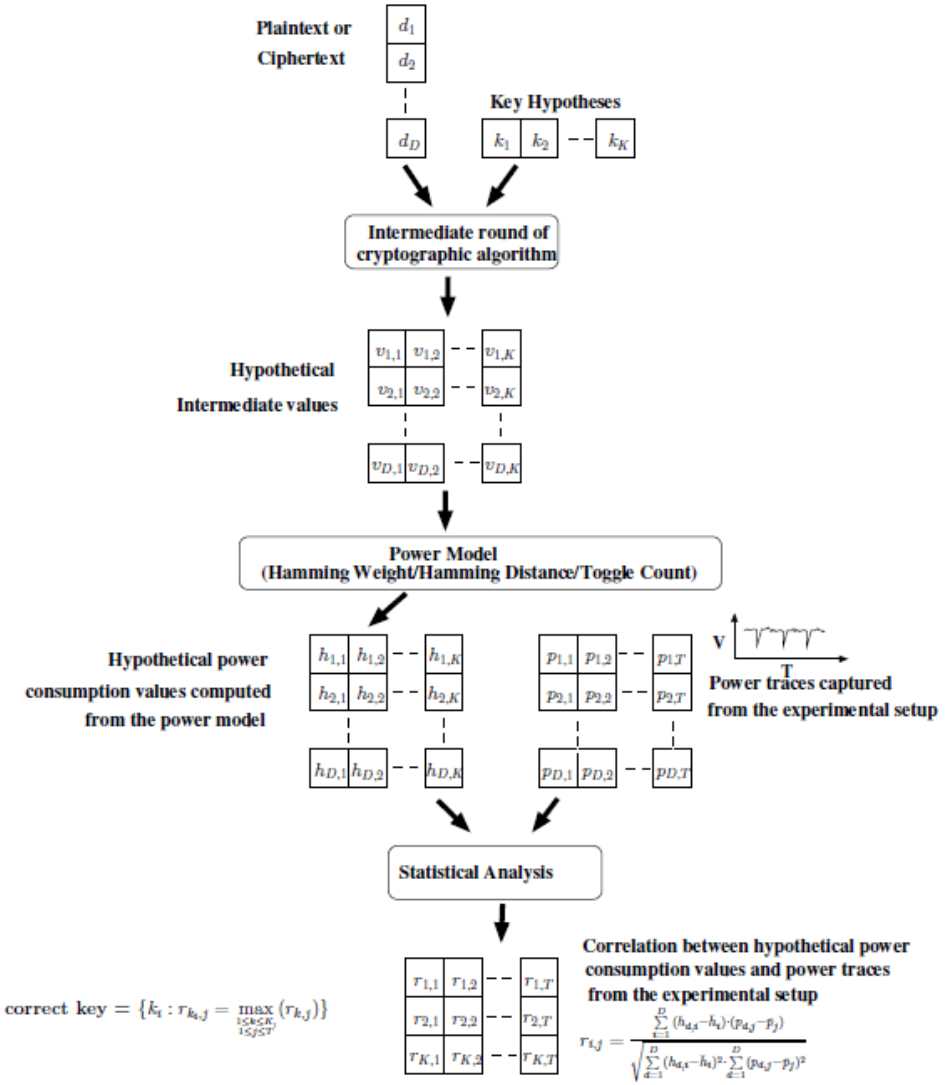




CPA on first key byte of 10th Round of AES
Power is Simulated by Hamming Distance of the Registers after each Round

Power is Simulated by Hamming Distance of the Registers after each Round, with superimposed Gaussian Noise.

Summary of Correlation Power Analysis



Metrics for SCA

- In order to evaluate the attacks and also to compare the crypto-implementations wrt. the SCAs we need quantifiable metrics.
- Useful Metrics:
  - Success Rate of a SCA adversary
  - Guessing Entropy of an Adversary

Success Rate of an Adversary

- SCA works as a divide and conquer strategy, where the key space is divided into several equivalent classes.
- The attack normally does not distinguish between keys which belong to the same class or partition.
- We can formalize the cryptographic implementation as $E_K,$ where K is the key space.
- The adversary assumes a leakage model, denoted by L
  - The leakage model provides some information about the key or some other desirable information.
- The adversary is an efficient algorithm denoted by $A_{E_K,L}$: bounded by time complexity, τ, memory complexity, $m,$ and q queries.

Formal Definition of Success Rate

- The leakage exploited by the adversary , maps a key $k \in K$, to a set, within which it cannot distinguish.
- We define these partitions by S, and the mapping by a function γ, st. $s = \gamma(k),\ k \in K$.
- Typically, $\left|S\right| \ll \left|K\right|$.
- The objective of the attack is to determine the class s to which a target k belongs with non-negligible probability.
- As an analogy, consider the Hamming Weight class, which divides the key space into equivalence classes or partitions.

Formal Definition of Success Rate

- As an analogy, consider the Hamming Weight class, which divides the key space into equivalence classes or partitions.
- The output of the adversary is based on the ciphertexts (black-box information) and the leakage (side channel information).
- The output is a guess-vector, which are the key classes sorted in terms of descending order of being a likely candidate.
- Thus we can define a order-o ($o \leq |S|$) adversary when the adversary produces the guessing-vector as $g = [g_1, g_2, \cdots, g_o]$

Formal Definition of Success Rate

Input: K, L, E_K
Output: 0 (failure), 1 (success)

1 $k \in_R K$
2 $s = \gamma(k)$
3 $\mathbf{g} = [g_1, \ldots, g_o] \leftarrow A_{E_k, L}$
4 **if** $s \notin g$ **then**
5 **return** 0
6 **end**
7 **else**
8 **return** 1
9 **end**

The experiment is a success if $s \in g$

$$Succ_{A_{E_K,L}(\tau,m,q)} = \Pr[Exp_{A_{E_K,L}} = 1]$$

Guessing Entropy of an Adversary

- The above metric for an o^{th}order attack implies the success rate for an attack where the remaining load is o-key classes.
- The attacker has a maximum of o-key classes to which the required k may belong.
- While the above definition is fixed wrt. the remaining work load, we define guessing-entropy to provide a more flexible definition for the remaining work load.

Formalizing Guessing Entropy

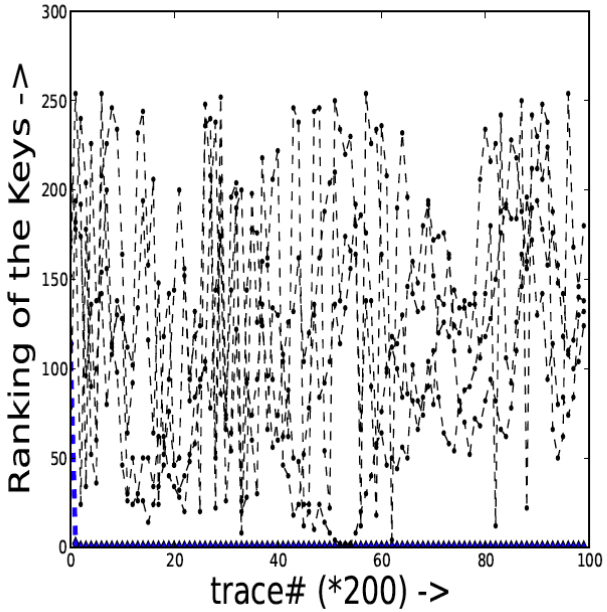
Input: K, L, E_K
Output: Key class i

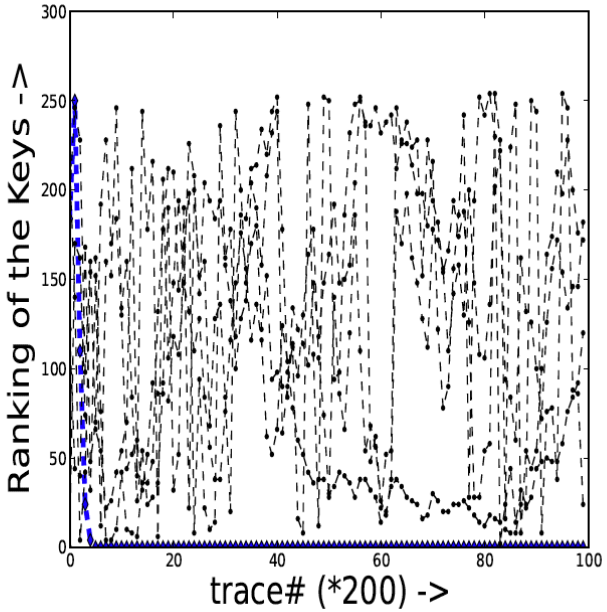
1 $k \in_R K$
2 $s = \gamma(k)$
3 $\mathbf{g} = [g_1, \ldots, g_o] \leftarrow A_{E_k, L}$
4 **return** i such that $g_i = s$

$$GE_{A_{E_K,L}(\tau,m,q)} = E[Exp_{A_{E_K,L}}]$$

- Formal definition of Guessing Entropy of the adversary against a key class variable S

Guessing Entropy Plots





CPA on Simulated Power Traces with and without noise.
Blue indicates that the guessing entropy for the correct key byte drops to 0 faster than the wrong keys.

CPA on Real Power Traces

- When attacking real power traces, the effect of electrical and algorithmic noise comes into effect.
- Architecture often has a strong effect on the algorithmic noise.
- It determines the Signal-to-Noise Ratio (SNR) which quantifies the quality of the power traces.
- This in turn affects the Success rate and the Guessing-Entropy of the attacks.

SNR of a Power Trace

- Consider a Cipher, like AES, with r-rounds and let S be a random variable representing the key dependent intermediate variable of E.
- S is called the target and satisfies $S = F_{k^*}(X),$ where X is a random variable representing a part of known plaintext or ciphertext.
- F_{k^*} is a function which depends on a part of the cipher and the output depends on a part of the secret key, denoted as k^*.
- The function F_{k^*} also depends on the leakage function of the hardware device.
- This component is also denoted as

$$L_t = aF_{k^*}(X) + N_t = aS + N_t$$

Here a is some real constant. N_t is some Gaussian Distribution $N_t \sim N(0, \sigma_t)$

Distinguishers: Univariate Vs. Multivariate

- Leakage has a deterministic part *aS.*
  - *Can be simulated by the adversary knowing the value of X and each key hypothesis.*
- Distinguisher*:* A mathematical function that the attacker applies at each point of the trace to distinguish the correct key from the wrong key.
- If the observed leakage is denoted by $F_k(X)$ then the distinguisher is a vector $D_t = \{d_k \,|\, k \in K\}_t$, where $d_k = D(aF_{k*}(X) + N_t, F_k(X))$
- So far, we have discussed distinguishers at one point t.

$$L_t = aF_{k^*}(X) + N_t = aS + N_t$$

Here a is some real constant. N_t is some Gaussian Distribution $N_t \sim N(0, \sigma_t)$

Distinguishers: Univariate Vs. Multivariate

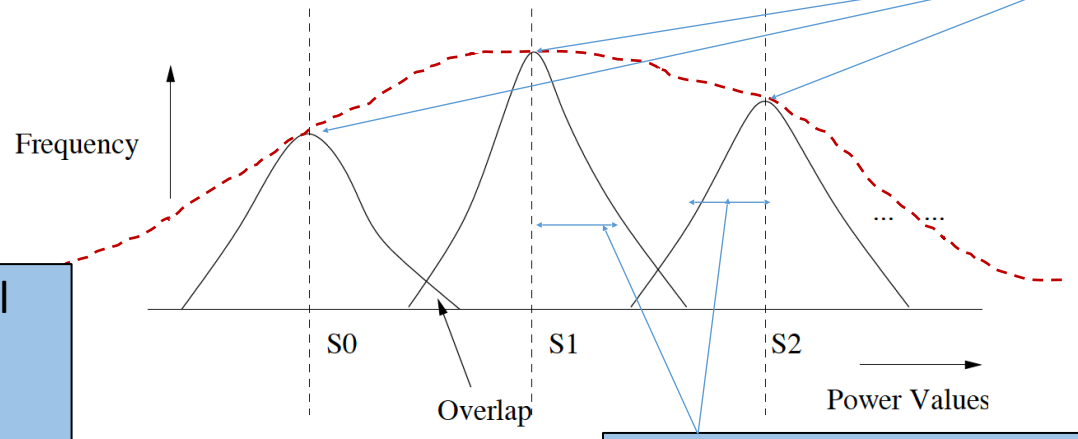
- So far, we have discussed distinguishers at one point t.
- But multiple trace points can be combined within a distinguisher.
  - Generally, done by calculating higher-order moments:
    - $(x - \bar{x})(y - \bar{y})$, where x, y are points on the trace at time t1 and t2
    - This is basically co-variance!!!
- Multivariate attacks are good..but
  - Both trace points have some noise, (some SNR)
  - The noise amplifies along with the signal.
  - So, have to be careful

Definition of SNR

- A register X is being updated by an AES circuit depending on the inputs.
  - We observe the power traces by varying the inputs and creating a frequency plot at some point of interest, t
  - Our power model tells that the power values would depend on the Hamming weights, say S

$Var(E[L_t|S])$ indicates the variations in leakage due to the target S at sample point t .

This contributes to the signal component, in aiding the attack.
More this value, the S-values are distinguishable better



$E[L_t|S]$

$Var\left(L_t - E[L_t|S]\right)$
contributes to noise.
More this variance, more is the overlap between the leakage due to the target variable S.
This hinders the attack.

$L_t - E[L_t|S]$

Definition of SNR

- $$SNR_t = \frac{Var(E[L_t \mid S])}{Var(L_t - E[L_t \mid S])}$$
  - The SNR is a very useful metric for assessing the threat of a power attack on a given implementation.
- How can it be computed in experiments?
  - Say we collect 10,000 traces.
  - We target a byte, say denoted as S.
  - Depending on the Hamming Weight (say) we split the traces into 9 bins.
  - At a time t, thus the i^{th}-bin consists of power values, $P_i^0, \cdots, P_i^{n_i-1}$
    - The average of this gives a point in the distribution $E[L_t \mid S]$. We calculate the variance for the signal.
  - From each of the power values in the i^{th}-bin we subtract the average of the bin.
    - The variance of this gives the noise component.

Relating Correlation with SNR

- $L_t = P_{det} + N,$ where P_{det} is the deterministic component of power and N is the noise.
- Thus the correlation between the total leakage L_t and the hypothetical power value for the i^{th} key H_i as $Corr(H_i, L_t)$:
- $$Corr(H_i, L_t) = Corr(H_i, P_{det} + N) = \frac{Cov(H_i, (P_{det} + N))}{\sqrt{Var(H_i)\left(Var(P_{det}) + Var(N)\right)}}$$
- Thus, $$Corr(H_i, L_t) = \frac{E\left(H_i . (P_{det} + N)\right) - E(H_i)E(P_{det} + N)}{\sqrt{Var(H_i)Var(P_{det})}\sqrt{1 + \frac{Var(N)}{Var(P_{det})}}}$$

Relating Correlation with SNR

- If H_i and N are independent,

  $Cov(H_i, \mathrm{N}) = 0 \Rightarrow 0 = E(H_i . N) - E(H_i)E(N) \Rightarrow E(H_i . N) = E(H_i)E(N).$

- $\therefore E\left(H_i . (P_{det} + N)\right) - E(H_i)E(P_{det} + N) = E(H_i .$

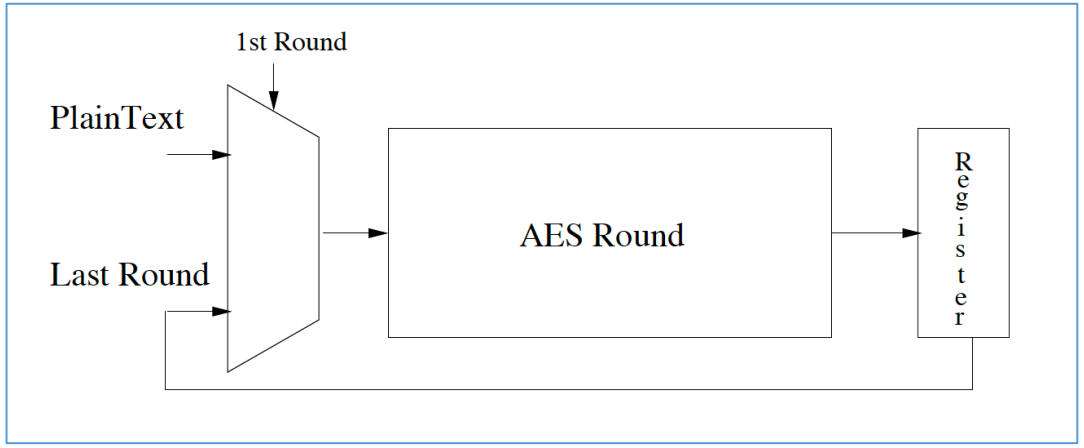
  $P_{det}) + E(H_i . N) - E(H_i)E(P_{det}) - E(H_i)E(N) = E(H_i . P_{det}) - E(H_i)E(P_{det})$

- Thus,

$$Corr(H_i, L_t) = \frac{E(H_i . P_{det}) - E(H_i)E(P_{det})}{\sqrt{Var(H_i)Var(P_{det})}\sqrt{1 + \frac{Var(N)}{Var(P_{det})}}} = \frac{Corr(H_i . P_{det})}{\sqrt{1 + \frac{1}{SNR}}}$$

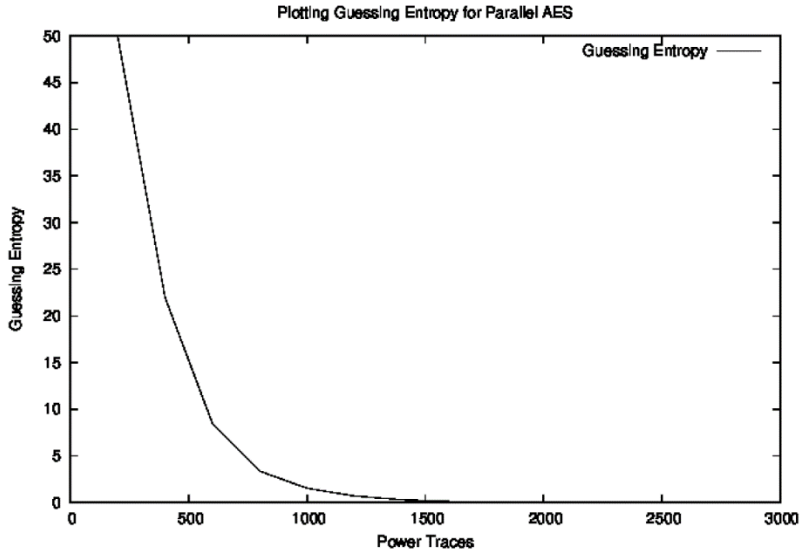
The above result can be used to evaluate an architecture based on simulated power traces. The simulation computes the value of $Corr(H_i . P_{det})$. However, the real correlation can be computed using the SNR.

DPA on an Iterative Architecture with Parallel S-Boxes

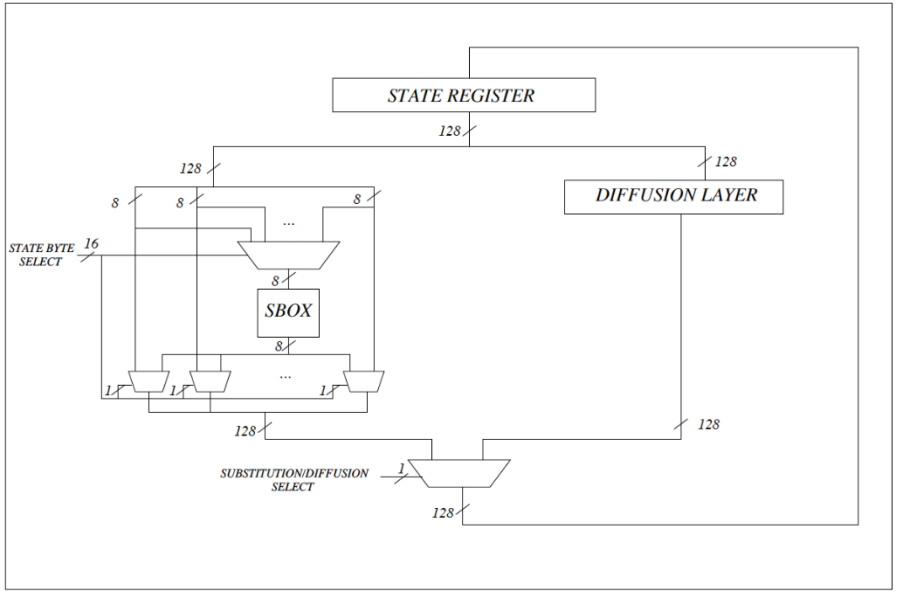


Power traces are collected and correlated with hypothetical power values to perform CPA.
All the S-Boxes are in parallel in this architecture, providing algorithmic noise.

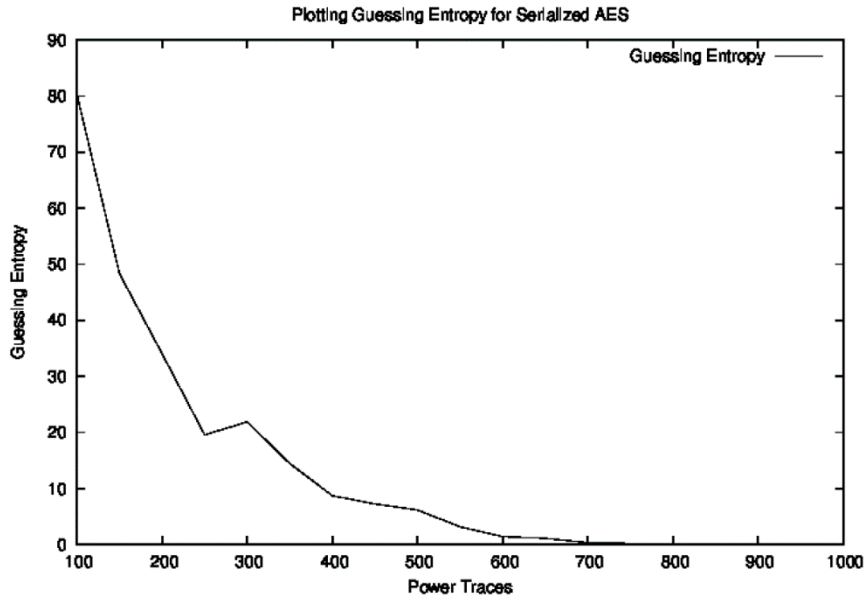
We acquired 70,000 power traces and divided into sets of 3000 traces each. We perform the CPA attacks on them, and plot the average Guessing Entropy for 3000 traces.



DPA on a Serialized Architecture of AES



- Serialized architecture, wherein there is only one S-Box.
- A MUX is used to select out one byte from the 128-bit state and process by the S-Box.
- Less algorithmic noise.



Plot of average Guessing Entropy for 1000 traces, averaged over 40 sets.
Compared to the parallel architecture, one can see the less number of observations required.

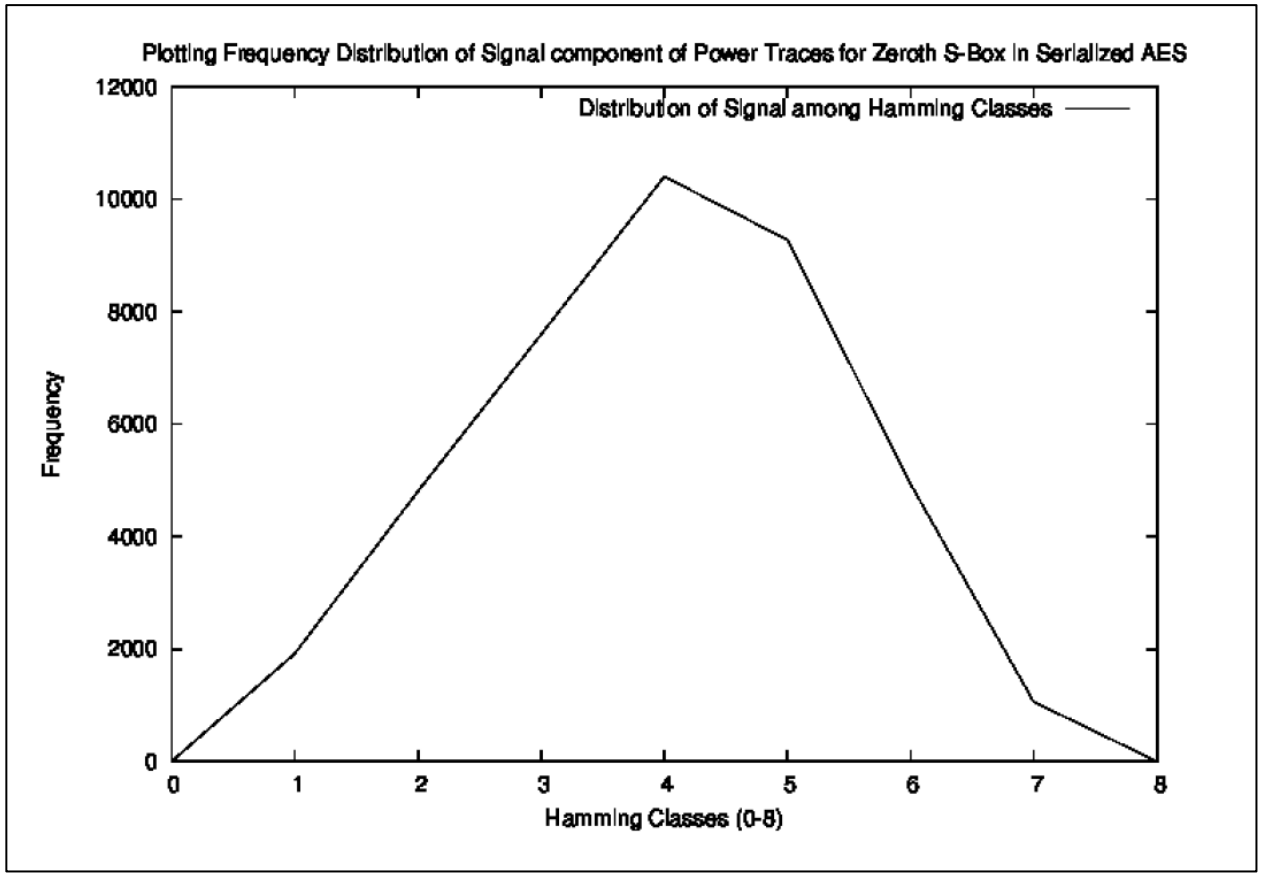
Shuffled Architecture and SNR

- A very popular technique to increase the resistance against CPA is called shuffling.
- In this architecture, the shuffling scheme randomizes the sequence of S-Box operations.
- Does not have a negative effect on throughput, as in when we defend using dummy operations.
- Quite easy to implement: make the select line in the multiplexer arbitrary.
- However, if one observes the total power in 16 clock cycles, one can still get the CPA working!

Comparing based on SNR Computations

- Serialized Architecture:
  - We observe the power consumption say during the 0th S-Box computation.
  - We compute the Hamming Distance (or weight) corresponding to the 0th S-Box. Note for this we need the key.
    - Classes can be from 0 to 8.
  - We calculate the average of each class: p_i, for $0 \leq i \leq 8$.
  - The number of elements in each class is say f_i .
  - Thus, Signal is $Var\left(E\left[L_t \middle| S \right] \right) = \frac{\Sigma_{i=0}^{8} f_i p_i^2}{\Sigma_{i=0}^{8} f_i} - \left(\frac{\Sigma_{i=0}^{8} f_i p_i}{\Sigma_{i=0}^{8} f_i} \right)^2$
  - For the noise component from each power trace, deduct the average p_i if the trace belongs to the i^{th} class. Calculate the variance across all the classes.

Frequency Plot of Signal Component of Power Traces for the 0-th S-Box of Serialized Architecture

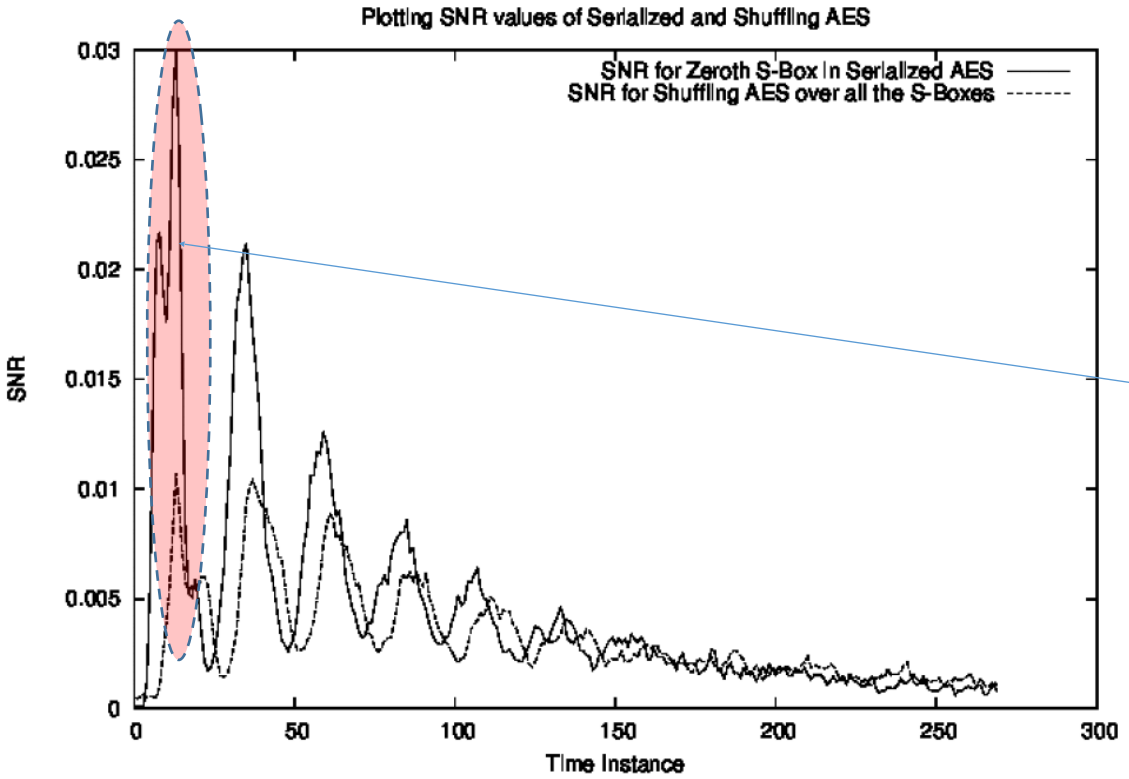


- The peak around 4 is expected.

SNR for Shuffled Architectures

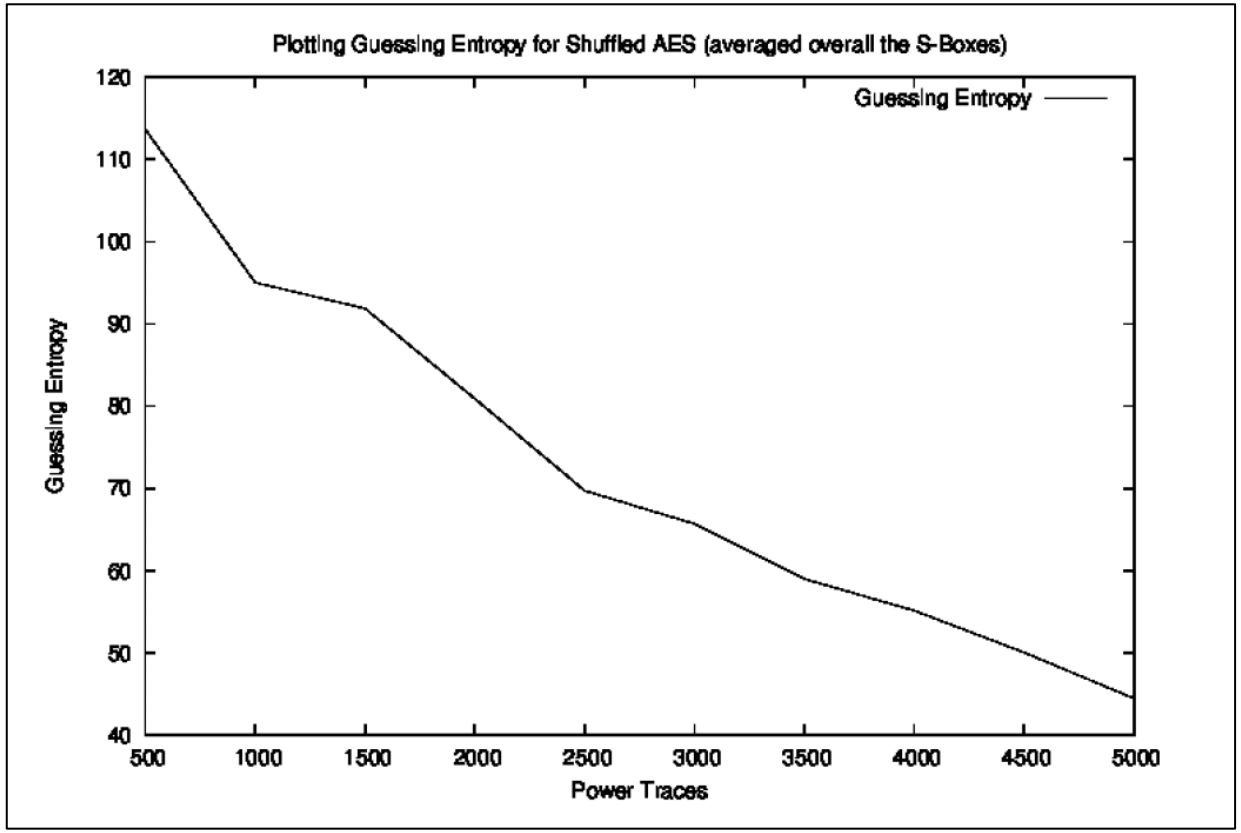
- For the shuffled scheme, we compute the SNR for the total power for all the 16 S-Boxes.
- This increases as if we target only one S-Box, then the SNR would be (1/256)-th that of the serialized implementation.
- We sum the power values in the traces for the sample points corresponding to each of the 16 cycles, and then they are divided into the Hamming Classes.
- It may be noted that since we are considering the total power for all the S-Boxes, the Hamming classes vary from 0 to 128.

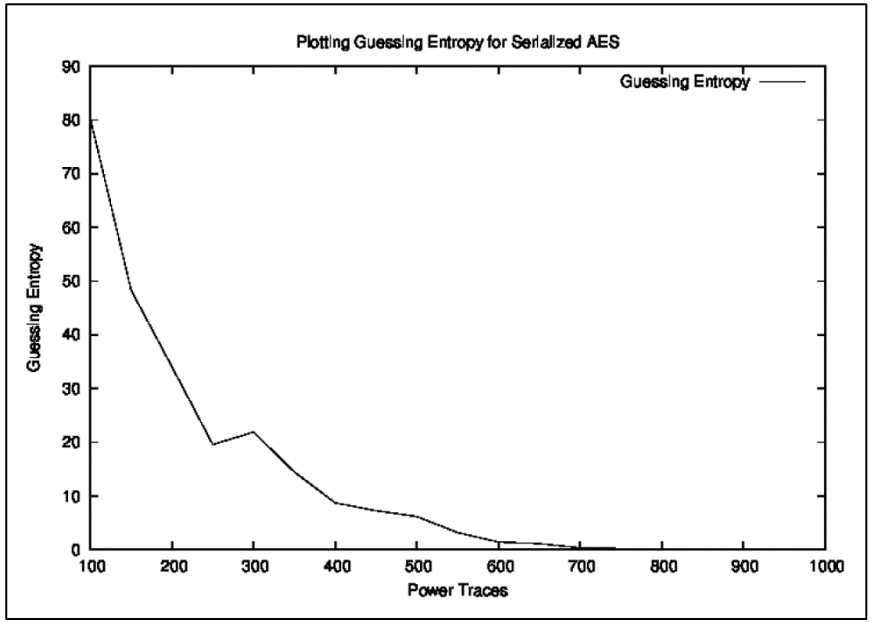
Comparison of SNRs



The average SNR for the shuffled scheme is around 1/3rd that of the serialized architecture.

Plot of Guessing Entropy for CPA on Shuffled vs Serialized Architectures





The GE reduces slowly compared to the serialized architecture

Profiled Attack

- The adversary has a device similar to the target device.
- Profiling Phase: It gathers a lot of traces with known key and models the noise and leakage model very well
- Attack Phase: Adversary collects a few (possibly 1) trace from the target device where the key is unknown and exposes the key
- This is also called template attacks
  - The profiles are called templates
- The modelling can be done very well with deep learning

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Exploiting Multiplie PoIs

- So far, we tried to use one PoI for our attacks
  - Sub-optimal approach
  - The information may occur at more than 1 point.
  - The measurement noise is also there
- Both the noise and signal can be correlated in multiple neighbouring points
  - We have to model all.
  - Trick: Multivariate normal distribution modelling of the noise.

Exploiting Multiplie PoIs

- Trick: Multivariate normal distribution modelling of the noise.
  - We shall have a mean vector and a covariance matrix

$$f(\mathbf{x}) = \frac{1}{\sqrt{2\pi \det(\mathbb{C})}} exp\left(-\frac{1}{2}(\mathbf{x}-\mathbf{m})^t \mathbb{C}^{-1}(\mathbf{x}-\mathbf{m})\right)$$

  - $\mathbf{x}$ is the input vector
  - $\mathbf{m} = [m_i]$, where $m_i = E[x_i]$ — mean vector
  - $\mathbb{C}$ is the covariance matrix. $\mathbb{C} = [c_{ij}]$, where $c_{ij} = Cov(x_i, x_j)$
    - $c_{ij} = c_{ji}$

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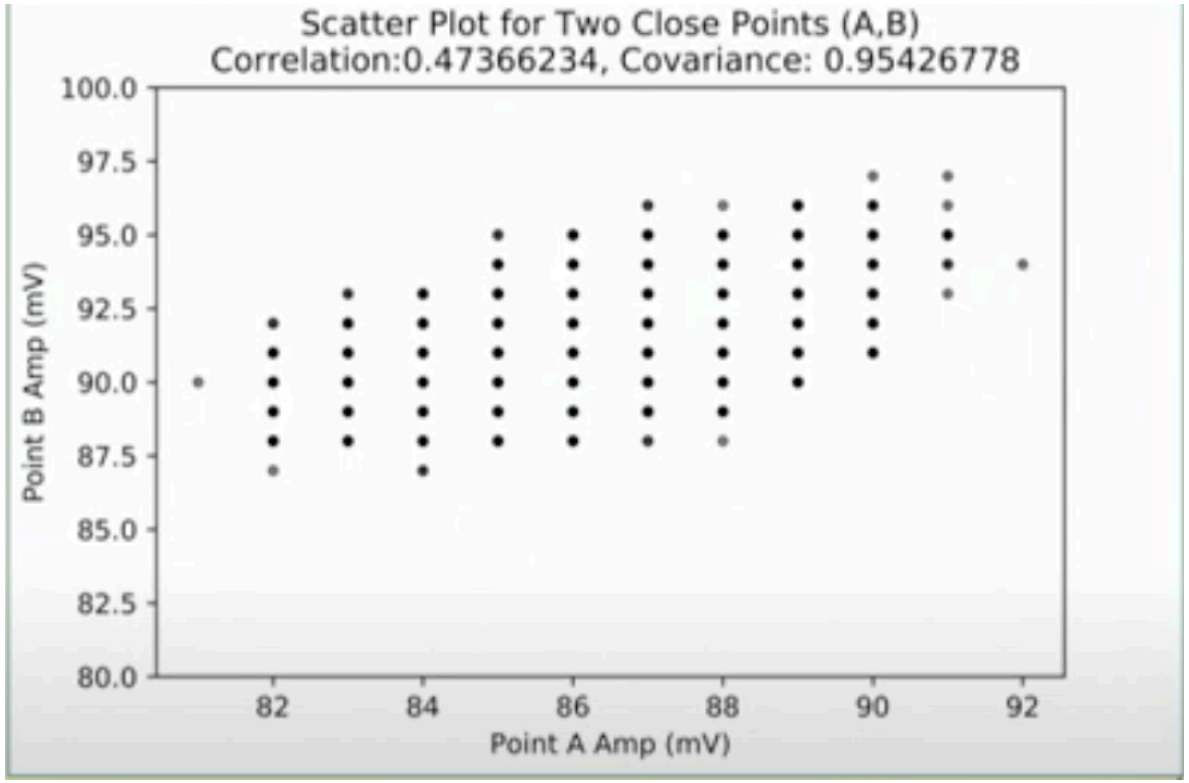
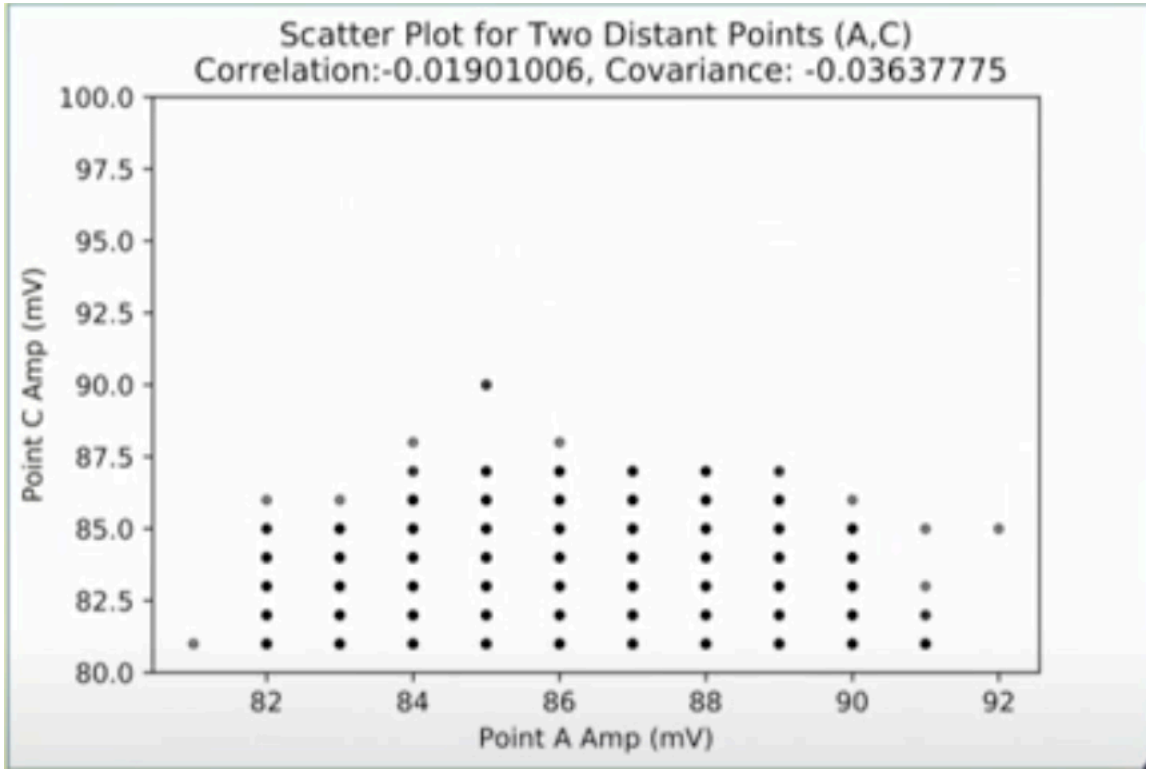
Correlated PoIs

- Electrical noise (and signal) are correlated in neighbouring points
- Can be visualized through scatter plot of correlation/covariances



Neighbouring points



Far away points

Building Templates for AES

- Key is known
- We go byte by byte as usual
- **Important**!!! We can target leakage from any point
    - Here we go with the leakage of S[i] = P[i] XOR K[i]
- We keep P[i] fixed and vary all other 15 plaintext bytes
    - We can also vary P[I] — more detailed template
- For each K[i]
    - We create leakage classes — based on some hypothetical leakage model
        - Say Hamming weight (HW)
        - But can also be the exact value of S[I]
- We also vary all other 15 key bytes randomly
- For **each value** of K[I], we have a template…

Building Templates for AES

- For each class,
  - Consider multiple PoIs from the trace (say n pods).
  - Calculate $\mathbf{m}$, and $\mathbb{C}$
  - $T_{P_i,K_i,j} = (\mathbf{m}, \mathbb{C})_{P_i,K_i,j}, 0 \leq j \leq 255$

Building Templates for AES

```
Mean Vector for Class 0 - Hamming Weight 0
[31.08695652 31.56521739 50.91304348 60.86956522 51.7826087  41.69565217 46.73913043 59.56521739 64.04347826
58.26086957]

Covariance Matrix for Class 0 - Hamming Weight 0
[[2.90118577 2.22134387 2.05335968 1.87549407 1.79249012 2.70948617 3.06916996 2.13043478 1.54150198 1.88537549]
 [2.22134387 4.16600791 4.5513834  2.80434783 1.44664032 2.63438735 4.10869565 3.62055336 2.20158103 1.98221344]
 [2.05335968 4.5513834  6.71936759 3.94268775 2.11660079 2.83596838 5.06719368 4.5513834  3.04940711 2.38735178]
 [1.87549407 2.80434783 3.94268775 3.39130435 2.06126482 2.00395257 3.46442688 3.21343874 2.64229249 1.94466403]
 [1.79249012 1.44664032 2.11660079 2.06126482 3.17786561 1.24901186 2.34980237 2.08300395 2.23715415 1.9229249 ]
 [2.70948617 2.63438735 2.83596838 2.00395257 1.24901186 4.03952569 3.82608696 2.36166008 1.65019763 1.94664032]
 [3.06916996 4.10869565 5.06719368 3.46442688 2.34980237 3.82608696 5.29249012 4.01778656 2.96640316 2.61660079]
 [2.13043478 3.62055336 4.5513834  3.21343874 2.08300395 2.36166008 4.01778656 4.07509881 2.83794466 2.11857708]
 [1.54150198 2.20158103 3.04940711 2.64229249 2.23715415 1.65019763 2.96640316 2.83794466 3.22529644 1.8972332 ]
 [1.88537549 1.98221344 2.38735178 1.94466403 1.9229249 1.94664032 2.61660079 2.11857708 1.8972332  2.29249012]]
```

Template for Hamming weight 0

Building Templates for AES

```
Mean Vector for Class 1 - Hamming Weight 4
[31.84227642 31.93333333 50.49430894 61.27100271 52.58373984 42.69376694 46.9696477  59.66233062 64.62547425
58.69322493]

Covariance Matrix for Class 1 - Hamming Weight 4
[[7.18172583 4.63698482 2.56824507 3.32747385 5.11652294 5.52488404 3.98544874 2.62838298 3.1274474  4.57197855]
 [4.63698482 6.28893709 4.90607375 3.3087491  3.15607375 4.61677513 5.11836587 4.1461316  2.97230658 3.07281273]
 [2.56824507 4.90607375 6.21214221 3.42182512 1.98136077 3.12217167 4.58062977 4.52220958 3.05345308 2.17721196]
 [3.32747385 3.3087491  3.42182512 4.18899059 3.39974516 3.1611261 2.97731895 3.07257846 3.18072824 3.1314466 ]
 [5.11652294 3.15607375 1.98136077 3.39974516 5.53270726 4.35076627 2.72597041 2.11912685 3.13902615 4.24913056]
 [5.52488404 4.61677513 3.12217167 3.1611261  4.35076627 5.72558477 4.1436285  2.88677701 2.89066863 3.94884603]
 [3.98544874 5.11836587 4.58062977 2.97731895 2.72597041 4.1436285 5.40254895 3.9040621  2.70934195 2.72549924]
 [2.62838298 4.1461316  4.52220958 3.07257846 2.11912685 2.88677701 3.9040621  4.47756615 2.77910546 2.15232057]
 [3.1274474  2.97230658 3.05345308 3.18072824 3.13902615 2.89066863 2.70934195 2.77910546 3.6042314  2.92734188]
 [4.57197855 3.07281273 2.17721196 3.1314466 4.24913056 3.94884603 2.72549924 2.15232057 2.92734188 4.45139117]]
```

Template for Hamming weight 4

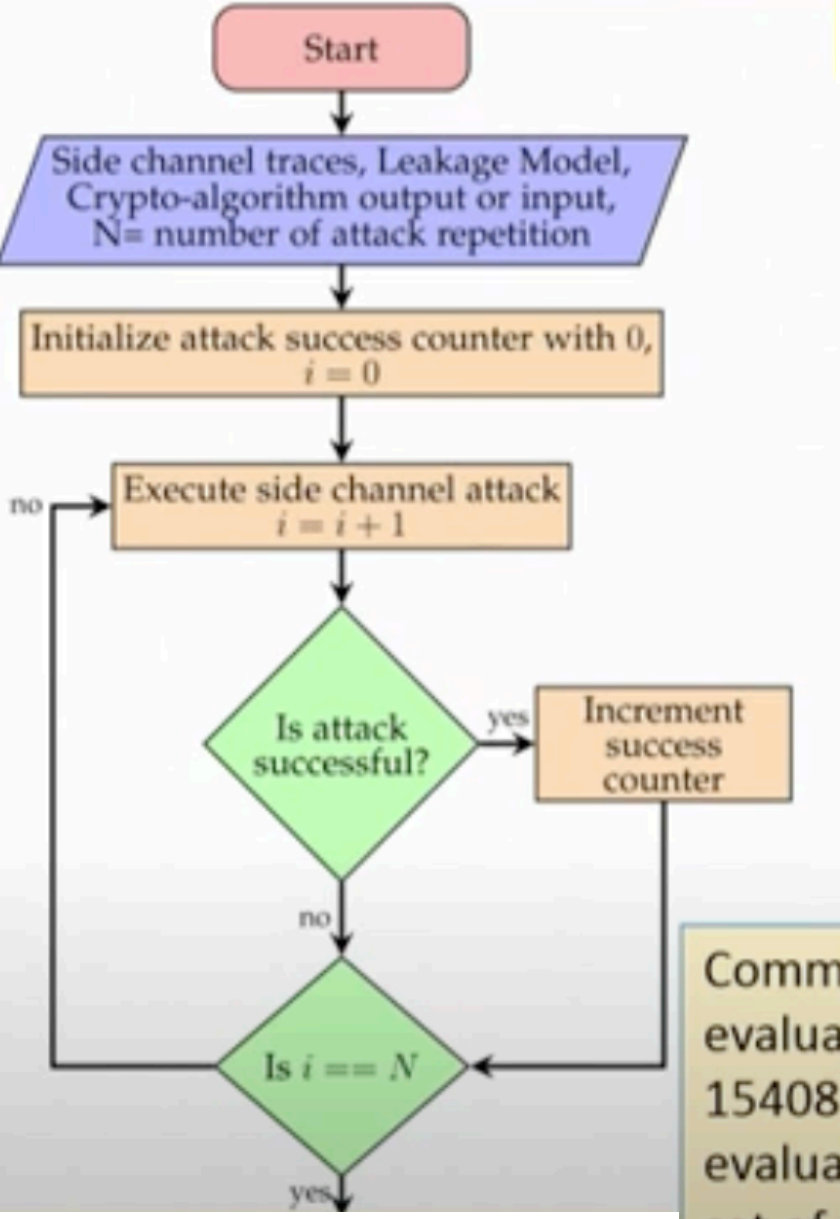
Attack Phase: Template Matching

- Get the traces from the target device — key unknown
  - You may keep a plaintext byte same, to keep the variation low
- Now, for each trace $\mathbf{t}$
  - Try to see which template it matches the best

$$Pr(\mathbf{t}; (\mathbf{m}, \mathbb{C})_{P_i,K_i,j}) = \frac{1}{\sqrt{2\pi \det(\mathbb{C})}} exp\left(-\frac{1}{2}(\mathbf{t}-\mathbf{m})^t \mathbb{C}^{-1}(\mathbf{t}-\mathbf{m})\right)$$

  - The template for which the **probability is maximum** gives the correct value of K[i]
  - **Important**: Do this for many $\mathbf{t}$, and **multiply** the probabilities so that you have more confidence
  - This is called **maximum likelihood estimation**
  - **Best Approach**: You can take log of probabilities and sum them — log likelihood estimate

Testing Side-Channel Attacks: Common Criteria



- Attacker performs different side channel attacks.
- Leakage Model is required
- Intermediate Value is required
- Non analytical methods to estimate Success Rate
- Dependent on lab expertise
- Time consuming and cumbersome
- Can quantify the vulnerability

Commom Criteria (CC) certification is an example of evaluation-style testing. Set of guidelines (ISO-15408) that defines a common framework for evaluating crypto-implementations using a standard set of predefined assurance levels.

Testing Side-Channel Attacks: Common Criteria

$$\mathrm{SNR} = \frac{\mathrm{Var}(\mathrm{E}(Y|X))}{\mathrm{E}(\mathrm{Var}(Y|X))}$$

$$\mathrm{NICV} = \frac{\mathrm{Var}(\mathrm{E}(Y|X))}{\mathrm{Var}(Y)}.$$

- Signal to Noise Ratio
  - Modified definition!!!
  - Actually the same
- Normalised Inter Class Variance (NICV):
  - New metric

- We denote by X and k a single plaintext byte and key byte,
- We also denote by $L = l(X, k)$ the normalized leakage model such that $\mathrm{E}(L) = 0$ and $\mathrm{Var}(L) = \mathrm{E}(L^2) = 1$.
- Note that we denote by Y the leakage measurement:

$$Y = \epsilon L + N$$

- where ϵ is the scaling coefficient and $N \sim \mathcal{N}(0, \sigma^2)$ is the noise component, which is independent of X.

SNR: Alternative Expression

- $Y = \epsilon L + N$
- Thus, $E(Y|X = x) = \epsilon l$, where $l = L(x, k)$.
- Note, that $E(Y|X)$ is a random variable and is a function of X, which takes the value $E(Y|X = x)$, at $X = x$.
- Thus, $E(Y|X) = \epsilon L \Rightarrow Var(E(Y|X) = \epsilon^2$
- Likewise, $Var(Y|X) = E(\left(Y - E(Y|X)\right)^2|X) = E[(\epsilon L + N - \epsilon L)^2|X] = E(N^2) = \sigma^2$
  - Note, $Var(N) = E(N^2) = \sigma^2$

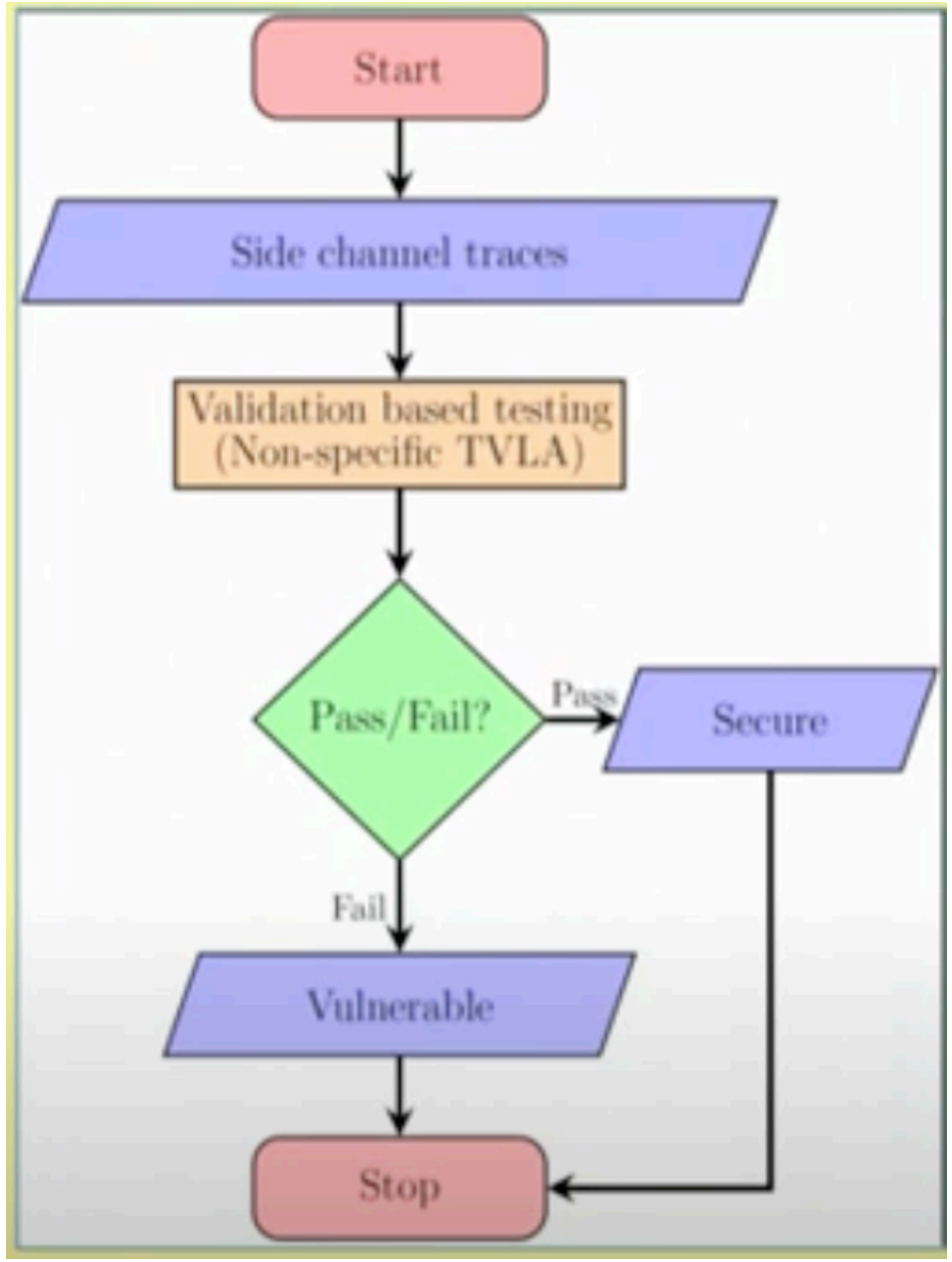
$$\text{SNR} = \frac{\text{Var}(\text{E}(Y|X))}{\text{E}(\text{Var}(Y|X))} = \frac{\epsilon^2}{\sigma^2}$$

NICV

- NICV is a technique which was designed to detect relevant point(s) of interest (PoI) in an SCA trace.
- The main advantage of NICV is that it is leakage model agnostic, and can be applied with the knowledge of only plaintext or ciphertext, and does not need the knowledge of target implementation or secret keys.

- $NICV = \frac{Var(E(Y|X))}{Var(Y)} = \frac{\epsilon^2}{Var(\epsilon L)+Var(N)} = \frac{\epsilon^2}{\epsilon^2+\sigma^2} = \frac{1}{1+\frac{1}{SNR}}$

FIPS: Validation Style Testing



- Does not require any knowledge of the leakage model
- Robust
- Does not depend on lab expertise
- Fast and easy to execute
- Fully analytical methodology
- Can *only* detect the presence of side channel leakage
- Fails to quantify the vulnerability

TVLA (Test Vector Leakage Assessment), proposed at NIST sponsored NIAT workshop in 2011 is one of the well known conformance tests.

FIPS certification is an example of conformance style testing that uses a cryptographic module

FIPS: Validation Style Testing — TVLA

- Test Vector Leakage Assessment (TVLA) is a direct application of Welch's t-test on side channel leakage traces for detection of vulnerabilities
- Non-specific TVLA: Partitions the trace based on public information, so can be useful for black box testing.
  - Acquire two sets of side channel traces
  - The first set corresponds to a fixed key and fixed plaintext
  - The second set contains traces corresponding to the same fixed key and random plaintext.

$$\widehat{\text{TVLA}}_x = = \frac{\left(\frac{1}{\sum_{q/x_q=x} 1} \sum_{q/x_q=x} y_q\right) - \left(\frac{1}{\sum_q 1} \sum_q y_q\right)}{\sqrt{\frac{1}{\sum_{q/x_q=x} 1}\left(\frac{1}{\sum_{q/x_q=x} 1} y_q^2 - \left(\frac{1}{\sum_{q/x_q=x} 1} y_q\right)^2\right) + \frac{1}{\sum_q 1}\left(\frac{1}{\sum_q 1} y_q^2 - \left(\frac{1}{\sum_q 1} y_q\right)^2\right)}}$$

Σ_q denotes $\Sigma_{q=1}^{Q}$ and $\Sigma_{q/t_q=t}$ denotes $\Sigma_{1 \le q \le Q, st. t_q=t}$

Asymptotically,

$$\widehat{TVLA}_x \to \begin{cases} \infty, if E(Y|X=x) \neq E(Y) \\ 0, otherwise. \end{cases}$$

$$t = \frac{\mu_0 - \mu_1}{\sqrt{\frac{s_0^2}{n_0} + \frac{s_1^2}{n_1}}} \quad \text{and} \quad v = \frac{\left(\frac{s_0^2}{n_0} + \frac{s_1^2}{n_1}\right)^2}{\frac{\left(\frac{s_0^2}{n_0}\right)^2}{n_0 - 1} + \frac{\left(\frac{s_0^2}{n_0}\right)^2}{n_1 - 1}}$$

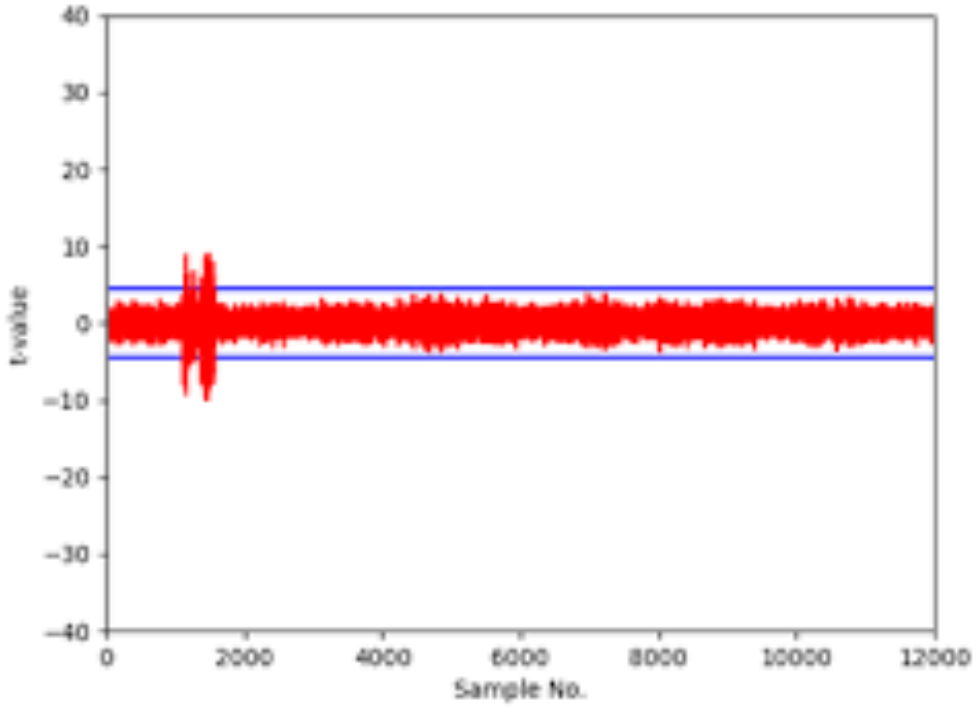
Non-Specific TVLA

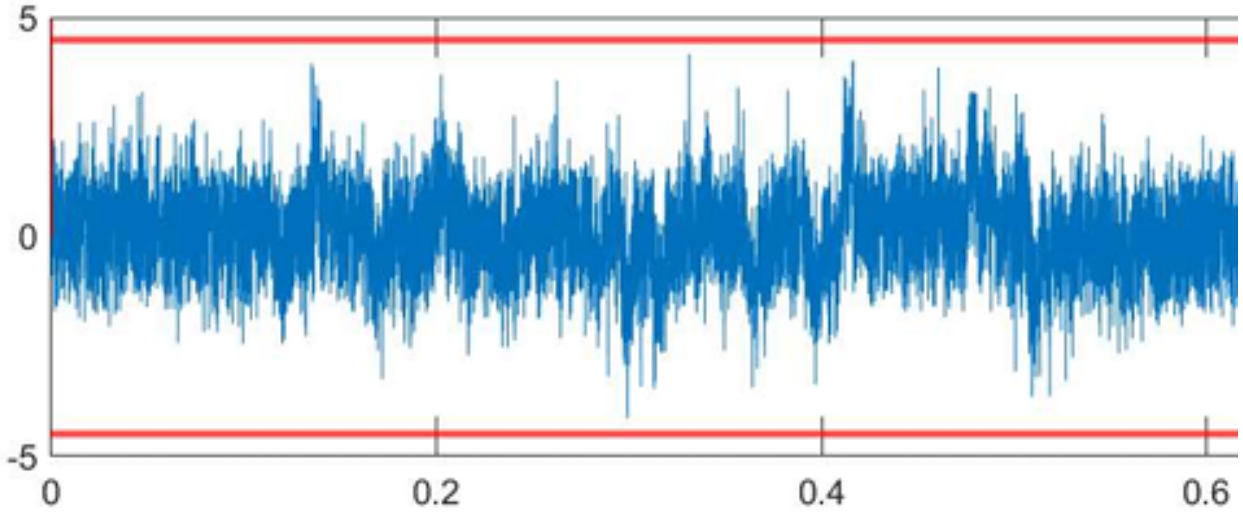
- $\widehat{TVLA_x} = \sqrt{Q}\,\frac{E(Y|X=x)-E(Y)}{\sqrt{Var(Y|X=x)+Var(Y)}}$
- We define, the asymptotic constant: $TVLA_x = \lim_{Q\to\infty} \frac{1}{\sqrt{Q}}\widehat{TVLA_x}$
- Thus, $TVLA_x = \frac{E(Y|X=x)-E(Y)}{\sqrt{Var(Y|X=x)+Var(Y)}} = \frac{\epsilon l(x,k)}{\sqrt{\sigma^2+\sigma^2+\epsilon^2}} = \frac{\epsilon l(x,k)}{\sqrt{2\sigma^2+\epsilon^2}}$

Specific TVLA

- Specific TVLA
  - Knowledge of secret key is required
  - The traces are partitioned depending upon some intermediate data of crypto-execution
  - Depending upon the choice of intermediate data, there could be multiple ways to do this partitioning
- A unified test methodology should bridge the gap between the validation style testing and evaluation style testing:
  - Specific TVLA can provide intuition on the source of leakage.
  - We use specific TVLA to extract more information regarding the side channel vulnerability of underlying crypto-implementations.
  - Extracted information can be used to estimate evaluation metrics like SNR, SR.

TVLA: How does it look?





X-axis in 10^3 scale

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